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Botany. — *On the occurrence of growth-accelerating and growth-retarding substances.* By CHRISTINE J. GORTER. (Communicated by Prof. F. A. F. C. WENT.)

(Communicated at the meeting of April 30, 1927).

As a result of his experiments, PAÁL (1919) became convinced, that substances are formed in the top of the Avena-coleoptile, which regulate the growth. In a normal coleoptile these substances are transported basipetally, and upon reaching the growth-zone, promote the growth, which means: the coleoptile grows straight.

When interrupted (injured) or exposed to the light at one side only, an uneven distribution of growth-regulators will result, and a positive curvature develops. PAÁL believes that light affects the growth-regulators, either bij destroying or retarding their formation, or by impeding their diffusion. PAÁL therefore is the first biologist to introduce the subject of growth-substances. His conception that these same substances have a relation to the conduction of the stimulus, is contradicted by many other experimenters.

STARK (1917—1922), claims, that certain specific stimulating substances play a role.

The experiments of WENT Jr. (1926) throw more light on the subject of the substances, which are formed in the coleoptile-top and which influence the growth. He has even isolated the growth-regulators from the coleoptile-tops.

Previous to these recent experiments Miss SEUBERT (1925) had endeavoured to find the above mentioned substances. She speaks of "Wuchsenzyme" where from we may deduct that she is of the same opinion as PAÁL. To become acquainted with these substances, we can follow two methods, miss SEUBERT says: either isolate the substances from the press-juice of the coleoptiles or determine which of "the various substances existing in the plant" are able to cause a curvature.

She found early in her experiments that it was impossible to isolate the active substances from press-juice. Therefore she investigated the influence of different substances upon the growth. She mixed the substance to be examined with agar, and put small blocks of this mixture on the top of the decapitated coleoptiles at one side.

When a negative curvature ensued (the covered flank of the coleoptile becoming convex) Miss SEUBERT concluded that at this side the growth had been increased.

The remarkable result was that the kind of curvature either positive or negative, was dependent upon the different concentrations of the same substance. Miss SEUBERT claims that the same substance either helps or retards the growth, depending upon the concentration.

My experiments have also shown that positive and negative curvatures are the result of the use of one substance in different concentrations ; but that at the same time, there is quite possibly a different explanation with regard to the development of a new physiological top.

DOLK (1926) has discovered, that a decapitated coleoptile reaches a growth-minimum after 150 minutes, and that thereafter the growth increases again by reason of the formation of new growth-accelerating substances ; which at the same time bring about a return of the phototropic sensibility.

Miss SEUBERT photographed the curvatures after 6 hours, "meist nach 6 Stunden wurde das Ergebnis notiert", and I repeated these experiments, keeping in mind however, the development of the new physiological top. A repetition of Miss SEUBERT's experiments does not seem to be superfluous, in as much as my results with pure agar differ from hers.

I describe below the experiments made with little blocks of unmixed agar.

The seedlings are sown in the usual way in a zinc box filled with soil, in a dark room of constant temperature (25° C.) and humidity (90 %). After decapitation and the loosening of the primary leaf, thirty minutes should elapse before the agar-blocks are placed on the tops. Only the perfectly straight seedlings were used.

It is necessary to loosen the primary leaf, because in continuing its growth, it might lift away the agar-block from the section, where it has been placed. The agar must be washed several times in both clear and distilled water, and then melted and poured on a glass plate. The mass is divided into small blocks and placed unilaterally on the top of the previously decapitated coleoptiles.

At regular periods the curvatures were observed. (See table I and II.)

During the first 2½—3 hours after decapitation the plants are still straight, but after some time a positive curvature develops, which steadily increases in intensity and after 5—6 hours the curvature is clearly visible. Three hours after decapitation more and more plants become positively curved.

It is evident, that there must be some relation between the positive curvatures and the newly formed physiological top. It is to be understood, however, that a positive curvature is not yet visible after exactly 2½ hours, because at that time the new growth-substances merely begin to form. Some considerable time must elapse before the different quantities of growth-substances at each side of the coleoptile appreciably affect the curvature. Moreover, in order to observe the first stages of the curvature, a much closer observation is needed. I have only taken in consideration curvatures that could be detected with the naked eye.

In regard to the influence of the agar-block upon the coleoptile, two conclusions may be drawn.

TABLE I (pure agar).

20 seedlings decapitated 12.30 h. Agar applied 1.15 h.

Time	Straight	Negative	Positive	Useless plants [1])
2.15 h.	20	0	0	—
2.45	20	0	0	—
3.15	18	0	2	—
3.45	12	0	7	1
4.45	7	0	12	1
5.45	4	0	14	2

TABLE II (pure agar).

22 seedlings decapitated 10 h. agar applied 10.30 h.

Time	Straight	Negative	Positive	Useless plants
11.30 h.	21	1	0	—
12.30	18	0	4	—
1.30	11	0	11	—
2.30	3	0	18	1
3.30	3	0	18	1
4.30	2	0	19	1

1. In the case of a plant which had been decapitated 3 hours before, growth substances are formed in the new top. At one side only a quantity of these substances diffuse upwards into the agar, so that less substances descend at that side, which causes a positive curvature.

2. No new top regenerates under the agar, but this is not the case at the other side: a positive curvature develops.

My results with unmixed agar differ from those of Miss SEUBERT. Miss SEUBERT found that the plants remained straight.

Some idea as to the intensity of the curvature, will be obtained from the picture below, which was taken by casting the reflexion of the plants on photographic paper.

These and many other experiments show that positive curvatures never develop within 3 hours after decapitation. Miss SEUBERT, investigating the influence of various substances upon the curvature, photographed the plantlets after 6 hours. If positive curvature resulted, one must consider that this was most likely due to the development of a new physiological top and not necessarily due to the growth-retarding influence of the substances.

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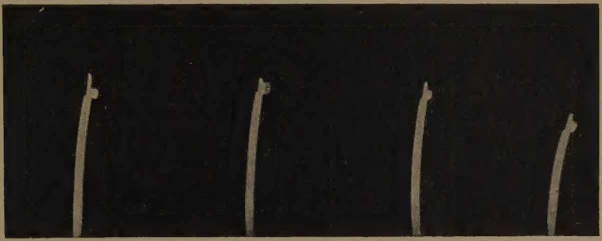
[1]) By useless plants I mean those upon which the agar-block dryed out or otherwise fell from its place.



Blocks of unmixed agar are placed on one side of the coleoptile, The picture, taken $5^1/_2$ hours after decapitation, shows the positive curvature quite clearly.

Therefore I experimented with many of the substances, used by Miss SEUBERT, and noted the time that elapsed before the curvatures developed.

Here again it was proven that positive curvatures never develop within 3 hours after decapitation.

A full description of my experiments would take too much space, and I will therefore merely mention that salts: KCl (concentration 10 % as well as 2½ %) did not give a positive curvature within 3 hours, after this period positive curvatures develop.

The conclusion is warranted, that KCl neither helps nor retards growth, but that the positive curvatures referred to by Miss SEUBERT are caused by the development of the new physiological top.

Using maltose (1 %) I had the same result. Miss SEUBERT obtained also a positive curvature by the use of various sugars. She therefore concludes: "Vergleichen wir die Wirkung der einzelnen Zuckerarten unter einander, so zeigt sich, dasz alle ungefähr gleich starke positive Krümmung erregen. Die Prozentzahl und die Stärke der Krümmung entspricht im Durchschnitt ungefähr dem der untersuchten Preszsaftkonzentration und dem einer $\frac{\text{mol}}{3}$ KCl- oder NaCl-Lösung."

She also found that independent of the nature of the substance or its concentration, the intensity of the positive curvature found, is about the same.

Even this fact should enable us to conclude that the curvature is caused by the new physiological top. Particularly since we know that a positive curvature does not begin to develop until after 3 hours after decapitation.

However there are different substances which cause negative curvatures, stimulating the growth (for instance several enzymes such as saliva, diastase, pepsine). These are the substances of which Miss SEUBERT found that they either helped or retarded growth, in accordance to the concentration.

I also experimented with these substances in different concentrations. Table III and IV show the results for diastase.

TABLE III (Diastase-agar 1 %).
14 seedlings decapitated 9.30 h. agar applied 10.30 h.

Time	Straight	Negative	Positive	Useless plants
12.— h.	0	14	0	—
1.30	0	14	0	—
2.30	0	14	0	—

TABLE IV (Diastase-agar 1 %, diluted 1 : 10).
41 seedlings decapitated 9.30 h. agar applied 10 h.

Time	Straight	Negative	Positive	Useless plants
12.— h.	31	10	0	—
1.—	18	3	16	4
3.30	23	2	12	4

The above tables show that:

1. Within 3 hours after decapitation negative curvatures develop by using diluted as well as undiluted diastase.

2. After about 3 hours positive curvatures begin to develop by using diluted diastase.

Here we find a tendency to bend in two different directions: the growth-accelerating action of the diastase will cause a negative curvature. This negative curvature becomes visible as soon as sufficient diastase has been diffused into the stump. The curvature will be more pronounced with increasing quantity of diastase diffusing into the plant. After 1½ hour these negative curvatures are very clearly visible. After 2½ hours the new physiological top begins to develop, and the plant shows an inclination to develop a positive curvature.

There are of course three posibilities:

a. The negative curvature is the strongest, completely dominating the positive curvature. This is the result of the use of great concentrations.

b. Both tendencies to curve are about the same. The plant first develops a negative curvature, and later straightens itself again.

c. The positive curvature is stronger than the negative. It sometimes happens that in the early stages the negative curvature is so slight that it is practically invisible, and the plant appears to be quite straight. Later on, however, the plant develops a positive curvature.

Another possibility is that a slight negative curvature forms immediately, and then later develops into a positive curvature.

In this case it will be observed that the plant is first negative, then straight and finally shows a positive curvature. This result has been obtained in many instances, thereby proving that diastase invariably tends to increase growth, but that the use of lower concentrations tends to make the negative curvature so slight, that after 6 hours it is completely dominated by the positive curvature.

The same is observed in the use of saliva and pepsin.

Miss SEUBERT's explanation of the growth-retarding influence of salts, sugars and diluted enzymes is the result of more or less complicated hypotheses. Therefore the idea that certain concentrated substances either help or retard growth, is unsound ; since we have shown that a positive curvature results from the formation of a new physiological top.

Summary: The substances used in my experiment all accelerate the growth. A growth-retarding substance has not been found. Its appearance should cause a positive curvature of the stump within 2½ hours.

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Physiology. — *On the Segmental Innervation of the Skin of Rays* [1]. By Dr. J. TEN CATE. (Communicated by Prof. G. VAN RIJNBERK).

(Communicated at the meeting of June 25, 1927).

The innervation of the skin of fish by the posterior roots of the spinal cord has thus far received little attention. Direct indications of a segmental innervation of the chromatophores of the skin by the sympathetic system in Pleuronectides are to be found in the researches by POUCHET [2]. VAN RIJNBERK [3] has established the segmental innervation of the skin by the pigmento-motor nerve fibres in Solea-, and Rhomboidichthys-species. VAN RIJNBERK [4] also demonstrated in sharks, in scillium catulus, and scillium canicula the segmental innervation of the skin by the posterior roots. This, to my knowledge, is about all that the literature contains concerning this problem.

I have conducted a series of experiments with rays (raja clavata), trying to make out how the skin of this fish is innervated by the posterior roots, and I applied principally the method of the so-called "remaining sensibility", suggested by SHERRINGTON for the determination of the boundaries of the dermatomata. As known, this method consists in isolating one dorsal root, or a group of them, by severing from two to four roots cranial or caudal to the first-named. Thus a sensible field is obtained, bounded by two insensible fields. By means of this demarcation of the sensible and insensible fields the location as well as the extent of the dermatomata can be established.

The spinal cord of rays, extending all along the vertebral column as far as the end of the tail, distributes a large number of dorsal and ventral roots to either side, which unite outside the vertebral column.

FÜRBRINGER [5] was the first to show that the first two or three spinal nerves are formed only by the ventral roots. In fair agreement with the observations by BRAUS [6] I could establish in most rays, examined by me, that already the third spinal nerve is formed as well by the ventral as by the dorsal root; but in a few cases the third spinal nerve was supplied only by the ventral root. In rays the first 17 or 18 spinal nerves form the plexus cervico-pterygialis, which in conjunction with the 18th to the 21st spinal nerve is located between the skull and the shoulder-girdle, and innervates the skin of the trunk before and above the shoulder-girdle, and of the anterior half of the pectoral as well. The spinal nerves emanating from the spinal cord behind the shoulder-girdle innervate directly the skin of the trunk and the hindermost half of the pectorals, without forming a plexus.

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[1] After the experiments made in the aquarium of the K. Zoöl. Genootschap N. A. M. at Amsterdam and the Zoöl. Station at Den Helder.

[2] cit. after V. RIJNBERK, Ergebnisse der Physiol., Bd. V, 1906.

[3] Archivio di Fisiologia, Vol. III, 1906.

[4] PETRUS CAMPER, Nederl. Bijdragen tot de Anat., D. III, Afl. 1, 1904.

[5] Ueber die spino-occip. Nerven der Selachier, Festschrift f. L. GEGENBAUER, III, Leipzig 1897.

[6] Jenaische Zeitschr. f. Naturwiss., Bd. **31**, N. F. **24**, 1898.

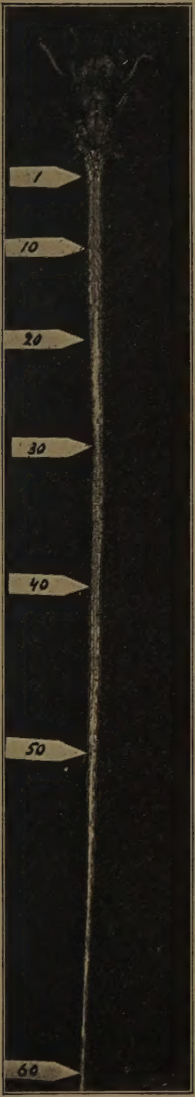

Fig. 1.

Generally the pectorals are innervated by the rami pterygialis of the 3d to the 40th spinal nerves. But I observed that also the 41st spinal nerve not infrequently participates in the innervation of the pectorals. Here, as well as in the formation of the plexus cervico-pterygialis, I could differentiate several variations.

Interpterygial nerves, such as in Squalides, do not occur in rays. The 41st or the 42d pair of the spinal nerves participates already in the innervation of the ventral fins. The last pair of the spinal nerves that innervates the ventral fins is the 56th; the next-following innervate the tail and form, as established by ROBIN, a plexus on either side, reaching as far as the end of the tail.

The boundary-lines and the location of the various dermatomata, identified by me after SHERRINGTON's method, I have indicated in a series of schemata. As, however the location as well as the boundaries of the dermatomata of the ventral side differ comparatively little from those of the dorsal side, I may be allowed to confine myself to reproducing only a general diagram of the boundaries of the dermatomata, and that of the dorsal side (Fig. 2).

It will be seen from this diagram that the course of the boundaries of the dermatomata differs. Nevertheless a prototype can easily be identified. The dermatomata lying on the level of the shoulder-girdle are most typical. They (from the 20th to the 22d) run fanlike from the diameter of the trunk to the margin of the pectoral fin and are broadening towards the periphery. More anteriorly and posteriorly this shape has more or less altered, because the part of the dermatoma that overlaps the fin has turned foreward or backward.

When measuring the breadth of these dermatomata at the dorsal and the ventral diameter, and at the lateral line of the trunk, where the skin of the trunk passes on to the fin, it is easy to convince oneself that the breadth of the dermatoma of the dorsal diameter gradually increases towards the lateral line. Near the ventral diameter the dermatoma is always broader than near the dorsal diameter, and about equal to, but frequently also a little broader than near the lateral line of the trunk. From the relations of the breadth of the dermatomata, just now referred to, it appears that in

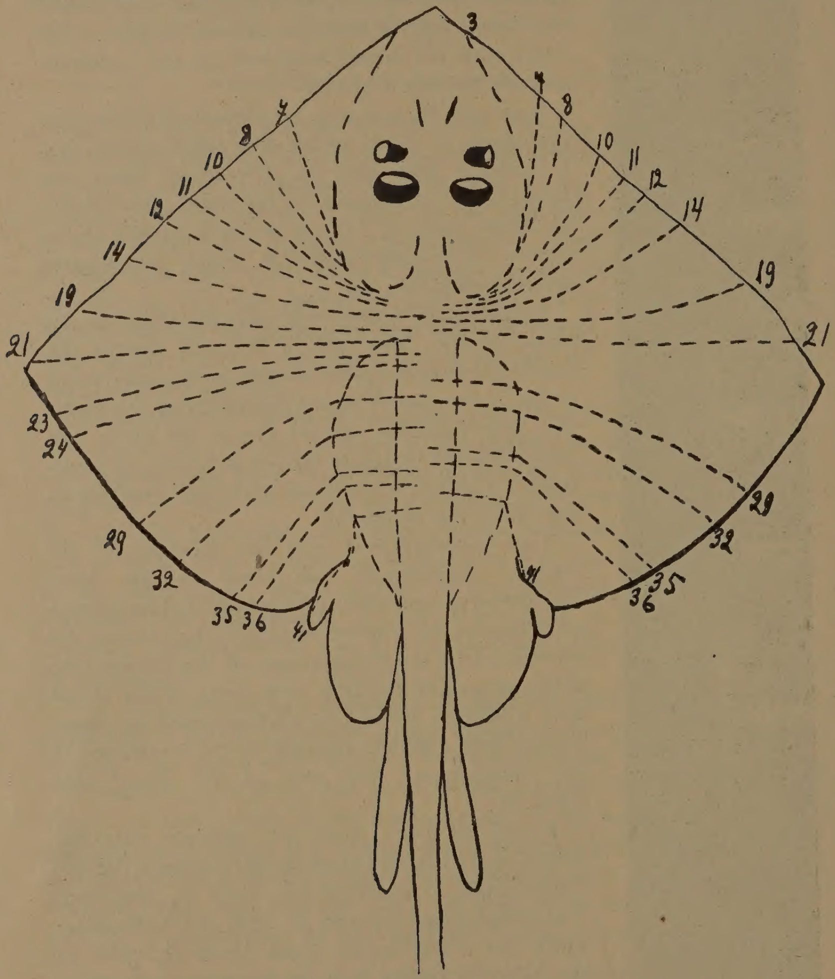

Fig. 2.
(The average boundaries of the dermatomata: to the left the caudal, to the right the cranial).

rays they are trapezoidal, as it is the case in most animals, and as has been described by VAN RIJNBERK also for sharks. The following diagram (fig. 3) is a fairly correct representation of the dermatomata in rays.

As stated above, we find in the fore-part of the body of rays a number of anomalies and complications in the structure of the dermatomata. The part of the trunk that is bounded in front by the head and laterally by the gill-pouches, is covered with a comparatively narrow strip of skin, which is innervated by the dorsal roots forming the plexus servico pterygialis. In this part of the skin I could not establish a distinct dermatomere. In a ray, however, of which the posterior roots of the 4th to the 10th spinal nerve had been cut across, I detected complete insensibility. More backwards, between the posterior margin of the gill-pouches and the shoulder-blade, on the other hand, the course of the dermatomata could well be made out. These dermatomata, also innervated by the plexus cervico pterygialis, run from the dorsal and also from the ventral side by way of the trunk to the margin of the fins in the form of the ribs of a fan. The peripheral part of these ribs, that overlaps the pectoral fin, reveals a larger deviation anteriorly the more cranially it is situated.

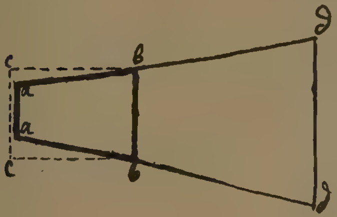

Fig. 3.

a—a dorsal diameter
b—b lateral line of the trunk
c—c ventral diameter
d—d margin of the fin

Quite a peculair position is occupied by the 3rd to the 9th dermatomata. Their whole course coincides with the anterior part of the pectoral fin, no contact with the trunk can be identified. The form of these dermatomata is also most like that of the ribs of a fan. The pointed end of these ribs is situated just at the head-region, the broad part coincides with the fin-margin. Here, no doubt, we have to do with a foreward displacement of

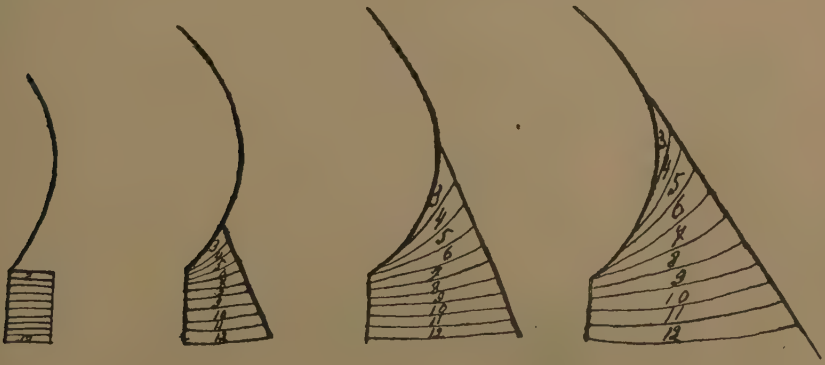

Fig. 4.

the dermatomata, which can be interpreted about in the same way as Bolk[1]) has suggested for the extremeties of mammals. In rays, however, the displacement of the dermatomata is only a foreward shifting, a fact that tallies quite well with Fürbringer and Braus's theory concerning the forward extension of the pectoral fin. The shifting of the dermatomata may be imagined, as is represented in Fig. 4.

The dermatomata that are situated on the trunk behind the shoulder-girdle spread on the posterior half of the chest in the shape of a fan. This form, however, has become different in that the peripheral part, that covers the fin, deviates the more posteriorly, the more the dermatomata are

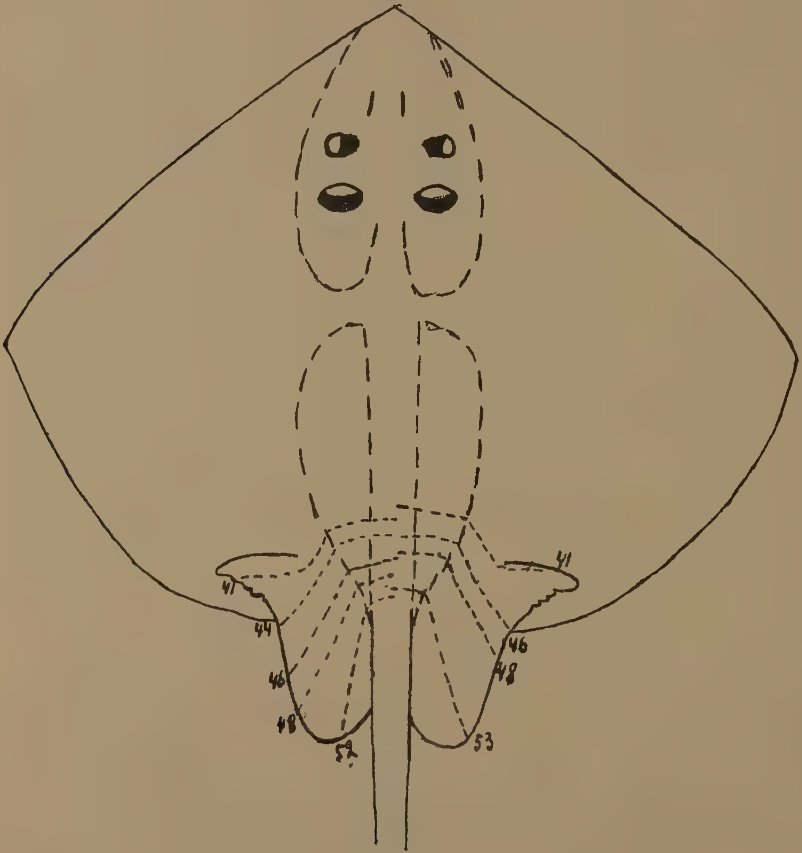

Fig. 5.

The average boundaries of the dermatomata: to the left caudal, to the right the cranial.

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[1]) De segmentale innervatie van romp en ledematen bij den mensch, Haarlem 1910.

disposed caudally, as may be seen in fig. 2. The more posterior the position of the dermatomata the smaller the angle becomes, at which the peripheral part of the dermatoma is inclined to the part of the trunk. The course of the dermatomata covering the trunk and the ventral fins is just the same. This is evident from fig. 5, in which I have given schematically the boundary lines of some of these dermatomata.

The innervation of the skin of the tail by the posterior roots happens in about the same way as on the trunk. In spite of the deviating configuration of the tail, the experiments after SHERRINGTON's isolation-method, and ROSSI's section-method, showed that the form of the dermatomata is the same here as on the trunk, namely a trapezium. This trapezoidal shape is here much more pronounced than on the trunk.

The overlapping of the dermatomata I have examined in some experiments after DE BOER's direct strychnine method ¹). As such experiments are comparatively difficult to perform with rays, I had to content myself in most cases with the indirect determination of the overlapping by the breadth of the sensible and insensible fields by the method suggested by WINKLER and VAN RIJNBERK ²).

These calculations have shown that in rays the overlapping in all parts of the body is larger than one third of the breadth of the dermatomata ; in most cases it is about one half of it. In some places, as e.g. on the ventral diameter the overlapping can be even more than the half of that breadth. The results obtained by the strychnine-method confirm the indirect measures found for the overlapping of the dermatomata.

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¹) Psychiatr. en Neurolog. bladen, 1918.

²) Versl. Kon. Ak. v. Wetensch., 1901.

Chemistry. — *The Flocculation of Agarsols through Salt Mixtures.* By Dr. E. H. BÜCHNER and D. KLEIJN. (Communicated by Prof. P. ZEEMAN.)

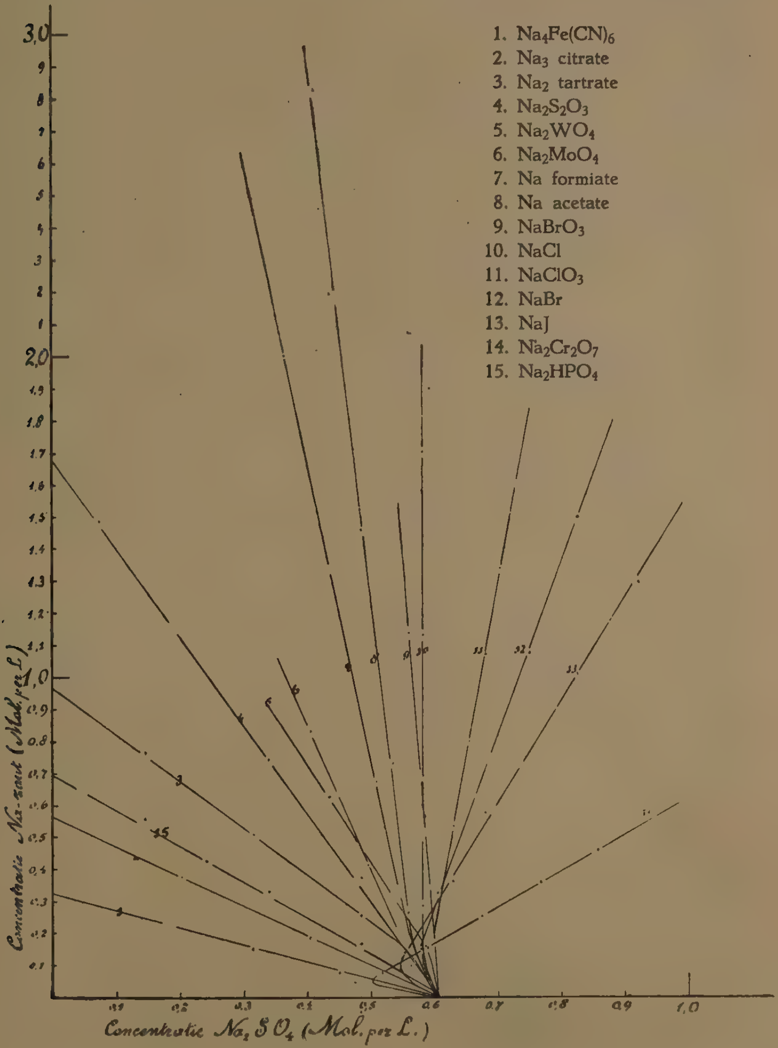
(Communicated at the meeting of June 25, 1927)

It is generally known that HOFMEISTER was the first to make systematic experiments on the flocculation of albumin from its solutions caused by different salts, in which he found that the salts can be arranged in the following succession according to their diminishing power of flocculation: citrate, tartrate, sulphate, acetate, chloride, nitrate, chlorate. Of a number of other salts, as bromide, iodide, rhodanide, HOFMEISTER perceived that no flocculation was obtained, not even when saturated solutions were used. Evidently these were not concentrated enough, and the flocculating power of these salts was still smaller than that of chlorate. This conception, according to which every salt should possess flocculating power, although this is sometimes not to be measured or even to be detected on account of its smallness, is as we believe, universally accepted. Nevertheless it appears to be erroneous, as we shall prove.

In an investigation of HOFMEISTER's series in agarsols the idea occurred to us to work with mixtures of salts in order to complete the series. When dealing with a salt that does not bring about flocculation itself, its solubility being too small, a quantity of a salt with flocculating power may first be added to the sol, but not so much that it should bring about flocculation. Then it may be determined how much of the first mentioned salt is still required to cause flocculation. If the two salts acted purely additively, the flocculation concentration of the apparently not flocculating salt might be calculated by extrapolation.

With this purpose a number of experiments were made in the following way. In a test tube 3cc. of a $1^0/_0$ agarsol were put, and in a second tube a solution of sodium sulphate and another salt in different proportions. This solution was filled up to 6 cc., the total volume, therefore, always being 9 cc. Generally three different concentrations were tried at the same time, the quantity of the salt solution always being increased by 0.1 cc., in such a way that the first did not flocculate, the second only feebly, and the third mostly very clearly. The experiments were made at about 50°; accurate regulation of the temperature was not necessary, as no change was to be observed even at 70°.

It now appeared that actually it is possible to speak approximately of an additive character of the flocculating power; a certain quantity of sodium sulphate may simply be replaced by another corresponding quantity of e.g. sodium tartrate. According as the flocculating power of the second salt decreases, a greater quantity of this salt is required to replace a definite quantity of sodium sulphate, in accordance with the expectation. Compare in the subjoined figure the lines for ferrocyanide, citrate, phosphate, tartrate, thiosulphate, tungstenate, molybdate, formiate, acetate, bromate. If, however, this experiment is made with sodium iodide as second salt, it appears that now *more* sodium sulphate is required to cause the agar to flocculate instead of less. Hence the



iodide counteracts the sulphate; in other words it does not only lack flocculating power, but it has even the opposite effect. Only in diluted solutions (less than 0.3 mol. NaJ) it has an influence in the same direction as sodium sulphate. The same behaviour as with iodide is met with in bichromate, perchlorate, bromide, chlorate, nitrate, and nitrite [1]), chloride behaving pretty well indifferently. Here, too, the action of the salts is additive; the lines in the graph are straight, at least starting from a certain concentration.

This result comes really to this that two groups of salts, flocculating ones and anti-flocculating ones, are to be distinguished [2]). This conflicts with the current conception, but it is in striking harmony with what has been observed in the swelling, in the gelatinisation and in the viscosity of colloids. There, too, iodide and sulphate are always found in opposition to each other; the first ion promotes swelling, lowers the setting point, diminishes the viscosity, the second has exactly an opposite effect in all these cases. Even more, it appeared in an investigation on the imbibition of gelatine in different salt solutions [3]) that the swelling was stronger than in water for chloride and nitrite (though very little), thiosulphate, bromate, chlorate, bromide, nitrate, perchlorate, iodide, whereas the other salts mentioned in this investigation oppose the swelling. Hence the groups of the flocculating and the imbibition opposing salts, of the anti-flocculating and the imbibition promoting salts almost completely coincide.

A full communication will later appear elsewhere. The experiments are being continued.

Inorg. Chem. Laboratory, University of Amsterdam,
High School, Oostburg.

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[1]) Not all drawn in the figure.

[2]) This division is not an absolute one, holding for all colloids. $NaClO_3$ e.g. still belongs to the flocculating group as far as albumin is concerned, but to the anti flocculating group as regards agar.

[3]) BÜCHNER, Rec. Trav. Chim. **46**, 439 (1927).

Physics. — *Provisional Results of Determinations of Gravity, made during the Voyage of Her Majesty's Submarine K XIII from Holland via Panama to Java.* By Prof. F. A. VENING MEINESZ.

(Communicated at the meeting of June 25, 1927).

Before making a communication, also on behalf of the Netherlands Geodetic Commission, of the provisional results of the determinations of gravity made on the oceans under the auspices of this commission, I will acquit myself of a debt of honour, though at this place I can not devote more than a few sentences to it. I wish to express my feelings of gratitude, first of all to the above-mentioned commission, and in particular to its president, Prof. Dr. J. J. A. MULLER, who has embraced the view, that though the investigation in question was outside the immediate sphere of action of the commission in a narrower sense, it yet did fall within its scope on a wider conception of its task as a scientific geodetic commission, and who on a liberal scale has made it possible for me to arrange and carry out the observations entirely according to my ideas.

Further I wish emphatically to express my indebtedness to the Dutch Navy, whose authorities have created the possibility of making these observations by giving me permission to travel on board the K II from Holland by the ordinary route to Java, and on board the K XI from Holland to Port-Said, and by finally crowning this work by directing the K XIII to make the long voyage to the Indies via the Panama Canal.

I also wish to render thanks, and this not in the last place, to the commanders, officers, petty officers and men of the boats, all of whom with the greatest readiness gave their assistance, and of whom I will only mention here as representatives Mr. L. A. C. M. DOORMAN and Mr. L. G. L. VAN DER KUN, resp. commanders of the K II and the K XIII.

And finally my thanks are due to the D. I. Radio Service, the Topographical Service at Weltevreden, and the Astronomical Observatory at Lembang, by whose energetic and efficient cooperation, with the support of Mr. K. A. R. BOSSCHA, an accurate radio time-service was organised, which supplied the time signals required for the observations during the crossing of the Pacific.

The discussion of the investigations is naturally divided into three sections :

1. The theoretic treatment of the method by which it is possible to carry out accurate determinations of the gravity at sea by means of pendulum observations.

2. The technique of the observations and the computations, and the new apparatus constructed after the experience got during the first voyage with the K II. During the voyage with the K XI it was put to the test, and

proved to be satisfactory; for the voyage in the K XIII it was further improved by cardanic suspension.

3. The choice of the route and the observation stations in connection with the problems, to whose solution contributions were expected, and the results obtained.

On this occasion only the third point will be discussed, and only the voyage of the K XIII will be treated. As regards the results of the voyage of the K II we may refer to the publications of the Netherlands Geodetic Commission:

"Observations de pendule sur la mer", 1923, and the subsequent:

"Isostatic reduction of the pendulum observations at sea", which gives the isostatic reduction of the observations, carried out disinterestedly by the bureau of the Coast and Geodetic Survey at Washington. The results of the voyage of the K XI, the isostatic reductions of which have likewise been computed by this bureau, will shortly appear.

For the results of the voyage of the K XIII Dr. WILLIAM BOWIE, Chief, Division of Geodesy, of the service mentioned, will also provide these reductions for which important contribution our sincere thanks may be expressed here. As however they have not yet all been computed, we prefer to give them in a subsequent publication, when the complete list can be given.

It was of great importance that when fixing the route of the K XIII the interests of the investigation were fully taken into account by the Navy. The following points came into consideration:

1. The investigation whether the longitude term in the normal formula for the gravity has any real significance. This term has made its appearance in the successive treatment of the gravity material on the continents by HELMERT (1915) and HEISKANEN (1925), as appears from the following formulae found by them:

$$g_0 = 978.052\,(1 + 0.005285 \sin^2\varphi - 0.000007 \sin^2 2\varphi + 0.000018 \cos^2\varphi \cos 2\,(\lambda + 17^\circ))$$

$$g_0 = 978.052\,(1 + 0.005285 \sin^2\varphi - 0.000007 \sin^2 2\varphi + 0.000027 \cos^2\varphi \cos 2\,(\lambda - 18^\circ)).$$

For the solution of this question the circle of observations surrounding the earth, made on the voyages of the K II and the K XIII, could give new material. It was desirable for this purpose that the voyage of the K XIII should not be made at too great latitude, and besides at latitudes that did not vary too much. For this reason the route across the Atlantic ocean was chosen via the Canaries to the West Indies, and across the Pacific, after San Francisco had been called at, which was desirable for navy-technical reasons, via Honolulu to Manila.

2. The investigation whether the isostatic equilibrium of the earth's crust, which save for local exceptions, has been found on the continents, is also present on the ocean.

3. The inquiry into the course of the gravity above the middle atlantic

ridge. For this purpose it was, of course, desirable to cross the lines of equal depth as much as possible at right angles. Once this crossing was already to take place on the route from the Canary Islands to the West-Indies. In order to get more material, however, it was resolved to go from Helder via the Azores to the Canary Islands, so that the ridge would once more be crossed.

4. The investigation of the course of the gravity above the great ocean deeps. During the voyage of the K XIII data could be obtained above four deeps:

the Porto-Rico deep, N. of Porto-Rico;
the Nero deep, E. of Guam;
the Yap deep, E. of Yap;
the Philippine deep, E. of the Philippine Islands.

Moreover during the stay in the Indies of the K XIII a detailed gravity investigation could take place above the deep South of Java, which region is also interesting because it plays such an important part for the Indies in seismic respect. The results of this investigation have not yet been provisionally calculated.

5. The investigation of the course of the gravity above the slopes of the continental shelves. A good opportunity was offered for this by the passage from Panama to San Francisco. Several times a course was taken towards the coast at about right angles to the lines of equal depth, in which a series of observations was made at the most suitable points.

6. The investigation of the course of the gravity in the Eastern part of the Malay Archipelago. Sailing from Manila via Strait Manipa and Ambon to Banda, and then from Banda along Dammar to the South to the Sahulshelf, the double arc of islands twice was crossed, during which observations could be made at all critical points.

From the Sahulshelf a course was taken round the North of Timor via Bima to Surabaya; on the way one observation was made, above the Flores deep.

As almost all the observations had to be made during submergence, the investigation required much of the crew: Sometimes the boat had to go down as often as four times a day, and a day without submarine practice was exceedingly rare. In every respect full attention was given to the interests of the investigation, and it may be stated with great satisfaction that the good spirits on board never suffered by it.

To complete the gravity investigations soundings have been carried out, for which the echo-method was applied. For the voyage of the K XIII the Navy had bought an echo sounding apparatus, which was, however, not entirely satisfactory; when sailing above water, e.g., no soundings were possible. The values obtained under water, which as regards place, correspond, therefore, with the determinations of the gravity, have, save for a few exceptional cases, probably no greater mean error than 15 m.

The purpose of the soundings was to obtain more data about the topography of the bottom for those parts where it was insufficiently known, and where this was not the case, to define the place of the gravity observations with respect to the topography of the bottom more accurately than was possible by the ordinary determination of the position of the ship.

The mean error of the gravity determinations, the results of which follow, may be estimated at ± 0.004 cm. The results, however, have only been provisionally computed, and are still unchequed; a definitive computation may modify them somewhat. The results have been reduced to sea-level, i.e. they represent the values which would have been found if the observations had been made at the surface of the sea.

The anomalies have been calculated with respect to three normal formulae:

Anomaly 1 with respect to the formula of HELMERT of 1901:

$$\gamma_0 = 978.030\,(1 + 0.005302\,sin^2\,\varphi - 0.000007\,sin^2\,2\varphi).$$

Anomaly 2 with respect to the formula of BOWIE of 1917:

$$\gamma_0 = 978.039\,(1 + 0.005294\,sin^2\,\varphi - 0.000007\,sin^2\,2\varphi).$$

Anomaly 3 with respect to the formula of HELMERT of 1915 without longitude term, which is the same as the formula of HEISKANEN of 1925 without longitude term:

$$\gamma_0 = 978.052\,(1 + 0.005285\,sin^2\,\varphi - 0.000007\,sin^2\,2\varphi).$$

The anomalies mentioned below in the text are the anomalies 2, i.e. those with respect to the normal formula of BOWIE.

The gravity stations have been marked on the adjoined map with the numbers of the stations.

PROVISIONAL RESULTS H. M. S. K XIII.

N⁰.	Date, 1926	Latitude φ	Longitude λ W. o. Greenw.	Echo depth meters	g_0 sea level	Anom. 1 0.001 cm.	Anom. 2 0.001 cm.	Anom. 3 0.001 cm.
1	2 June	44° 12′	15° 33′	5040	980.548	+ 4	— 1	— 10
2	3 „	43 12.8	18 46.3	4410	980.510	+ 56	+ 50	+ 41
3	4 „	41 33.4	21 37	4000	980.347	+ 42	+ 36	+ 27
4	5 „	39 48	24 57	3540	980.210	+ 62	+ 56	+ 47
5	6 „	38 31.78	28 37.45	Horta	980.162	+126	+120	+110
6	10 „	36 23	26 43	3580	979.895	+ 47	+ 41	+ 31
7	11 „	33 42.3	24 19	5450	979.637	+ 16	+ 10	0
8	12 „	30 46.3	22 29.8	5140	979.404	+ 21	+ 15	+ 4

PROVISIONAL RESULTS H. M. S. K XIII. (*Continued*).

N⁰.	Date 1926	Latitude φ	Longitude λ E. o. Greenw.	Echo depth meters	g_0 sea level	Anom. 1 0.001 cm.	Anom. 2 0.001 cm.	Anom. 3 0.001 cm.
99	31 Oct.	09° 30′.71	138° 10′.36	Yap	978.477	+303	+294	+281
100	2 Nov.	09 21.2	138 46.9	4500	978.190	+ 23	+ 15	+ 2
101	3 „	09 30.6	136 35.4	4770	978.204	+ 33	+ 25	+ 12
102	4 „	09 54.0	132 46.5	6080	978.213	+ 31	+ 22	+ 9
103	5 „	10 12.4	129 21.5	5760	978.256	+ 64	+ 56	+ 43
104	6 „	10 21.2	127 40.3	5760	978.265	+ 68	+ 59	+ 46
105	6 „	10 21.2	126 43.8	8740	978.012	—185	—194	—207
106	6 „	10 14.8	126 00.0	52	978.482	+289	+280	+267
107	8 „	13 19.0	121 37.7	541	978.336	+ 32	+ 23	+ 10
108	9 „	14 35.18	120 57.92	Manila	978.361	+ 4	— 5	— 17
109	20 „	08 50.4	121 52.2	4870	978.138	— 14	— 22	— 35
110	22 „	04 34.9	123 44.0	5140	978.157	+ 94	+ 85	+ 72
111	24 Nov.	—00 29.2	125 59.1	2380	977.825	—205	—214	—227
112	24 „	—01 44.7	126 57.0	1440	978.058	+ 24	+ 15	+ 2
113	24 „	—02 35	127 12	5180	978.051	+ 11	+ 1	— 12
114	25 „	—03 22.8	127 27.2	3540	977.999	— 48	— 57	— 70
115	25 „	—03 41.3	128 10.4	Ambon	978.180	+129	+120	+107
116	30 „	—03 59.0	129 23.0	4500	978.036	— 19	— 28	— 41
117	1 Dec.	—04 32.0	129 53.7	Banda	978.272	+210	+201	+188
118	2 „	—05 35.5	129 28.2	4840	978.123	+ 44	+ 35	+ 22
119	3 „	—07 10.3	128 54.0	3420	978.142	+ 32	+ 23	+ 10
120	3 „	—07 40.0	128 47.0	4600	978.038	— 84	— 93	—106
121	3 „	—08 13.3	128 32.8	1060	978.112	— 23	— 32	— 45
122	3 „	—08 48	128 26	2100	978.019	—132	—140	—153
123	4 „	—09 36	128 07	100	978.230	+ 57	+ 48	+ 35
124	6 „	—07 52.1	121 56.7	2570	978.151	+ 24	+ 15	+ 2
125	6 „	—07 44.7	119 58.5	4990	978.032	— 92	—101	—114
126	7 „	—08 27.1	118 42.7	Bima	978.274	+132	+123	+110
127	11 „	—07 53.2	114 53.8	900	978.190	+ 63	+ 54	+ 41
128	15-16 „	—07 12.11	112 44.63	Surabaja	978.138	+ 27	+ 18	+ 5

PROVISIONAL RESULTS H. M. S. K XIII. (*Continued*).

N⁰.	Date 1926	Latitude φ	Longitude λ W. o. Greenw.	Echo depth meters	g_0 sea level	Anom. 1 0.001 cm.	Anom. 2 0.001 cm.	Anom. 3 0.001 cm.
69	18 Sept.	22° 57′.4	153° 41′.1	4580	978.880	+ 65	+ 57	+ 45
70	19 „	22 14.1	155 24.9	4510	978.799	+ 30	+ 22	+ 10
71	19 „	21 45	156 15	5420	978.660	— 79	— 87	— 99
72	20 „	21 09	157 28	520	978.885	+183	+175	+163
73	21 „	21 18.45	157 52.0	Honolulu	978.942	+231	+223	+211
74	2 Oct.	20 48.8	158 36.5	4290	978.685	+ 3	— 5	— 17
75	3 „	20 27.7	160 50.3	4580	978.693	+ 32	+ 24	+ 12
76	4 „	19 58.0	165 02.6	4950	978.662	+ 30	+ 22	+ 10
77	5 „	19 30.7	168 28.2	3520	978.669	+ 63	+ 55	+ 43
78	6 „	19 06.7	171 34.8	2640	978.666	+ 83	+ 75	+ 63
79	7 „	18 39.1	174 57.7	1790	978.663	+105	+ 97	+ 85
80	8 „	18 06.0	178 13.5	3820	978.551	+ 22	+ 15	+ 3
81	9-10 „	17 46.8	E. o. Greenw. 179 33.6	4890	978.520	+ 9	+ 1	— 11
82	11 „	17 01.7	176 23.7	3180	978.543	+ 70	+ 62	+ 50
83	12 „	16 12.1	171 53.4	5600	978.442	+ 10	+ 2	— 10
84	13 „	15 32.2	168 25.7	5600	978.411	+ 10	+ 2	— 10
85	14 „	15 07.2	164 56.2	5330	978.409	+ 28	+ 19	+ 7
86	15 „	14 43.3	161 30.0	5490	978.362	— 1	— 10	— 22
87	16 „	14 05.0	158 09.5	5880	978.328	— 8	— 16	— 28
88	17 „	13 40.5	155 57.6	5910	978.310	— 8	| 17	— 29
89	18 „	12 41.7	150 56.5	5780	978.311	+ 32	+ 23	+ 10
90	19 „	12 01.0	147 36.9	5610	978.286	+ 32	+ 23	+ 10
91	20 „	12 15.0	146 00.9	6690	978.200	— 62	— 71	— 84
92	20 „	12 46.2	145 38.7	8740	978.057	—225	—234	—247
93	20 „	13 04.5	145 18.3	2850	978.400	+106	+ 97	+ 84
94	28 „	13 26.8	144 39.8	Guam	978.538	+229	+220	+207
95	29 „	13 41.5	142 52.7	3610	978.390	+ 71	+ 62	+ 49
96	30 „	10 35.0	140 22.2	2600	978.304	+100	+ 92	+ 79
97	30 „	09 56.8	140 04.7	2340	978.285	+101	+ 93	+ 80
98	31 „	09 23.3	138 31.3	7720	978.030	—138	— 146	—159

PROVISIONAL RESULTS H. M. S. K XIII. (*Continued*).

Nº.	Date 1926	Latitude φ	Longitude λ W. o. Greenw.	Echo depth meters	g_0 sea level	Anom. 1 0.001 cm.	Anom. 2 0.001 cm.	Anom. 3 0.001 cm.
39	3 Aug.	10° 22'.1	88° 27'.9	3470	978.255	+ 58	+ 49	+ 36
40	5 „	13 35.4	95 27.3	3870	978.366	+ 51	+ 42	+ 30
41	6 „	14 57.5	98 24.9	3660	978.416	+ 42	+ 34	+ 22
42	6 „	15 16.7	98 21.4	3970	978.408	+ 20	+ 11	— 1
43	6 „	15 37.6	98 17.8	4720	978.334	— 70	— 79	— 91
44	6 „	16 01.3	98 12.4	890	978.460	+ 37	+ 29	+ 17
45	8 „	17 30.6	103 26.4	5020	978.421	— 76	— 85	— 97
46	8 „	18 00.0	103 26.4	3050	978.453	— 70	— 78	— 90
47	8 „	18 14.1	103 27.4	710	978.567	+ 31	+ 23	+ 11
48	10 „	23 06.0	106 24.8	Mazatlan	978.861	+ 36	+ 28	+ 16
49	18 „	24 26.0	113 30.3	3610	978.925	+ 11	+ 4	— 8
50	19 „	26 31	115 40	3140	979.084	+ 25	+ 18	+ 7
51	19 „	27 01.0	115 21.0	3340	979.074	— 21	— 29	— 40
52	19 „	27 20.5	115 10.0	3080	979.060	— 60	— 67	— 78
53	20 „	29 18.7	117 05.0	3320	979.238	— 29	— 37	— 48
54	21 „	31 01	119 18	3760	979.398	— 3	— 10	— 21
55	22 „	33 11	121 30	4010	979.582	+ 5	— 2	— 12
56	23 „	35 50	122 42	3490	979.808	+ 7	+ 1	— 9
57	24 „	37 48.5	122 25.9	San Franc.	979.998	+ 26	+ 20	+ 10
58	8 Sept.	37 33.7	123 02.2	680	980.162	+211	+205	+195
59	9 „	37 08.7	123 53.8	3770	979.894	— 20	— 26	— 36
60	9 „	36 20.7	125 25.7	4530	979.842	— 3	— 9	— 19
61	10 „	34 46.4	128 33.7	4940	979.708	— 3	— 9	— 19
62	11 „	33 14.9	131 52.0	5130	979.583	0	— 7	— 17
63	12 „	32 12.4	134 00.1	4940	979.505	+ 8	+ 1	— 9
64	13 „	30 15.1	138 29.8	4830	979.349	+ 8	+ 1	— 10
65	14 „	28 57.3	141 15.8	4960	979.270	+ 30	+ 23	+ 12
66	15 „	27 27.8	144 20.9	4890	979.171	+ 43	+ 36	+ 25
67	16 „	25 46.2	147 42.7	5210	979.036	+ 30	+ 23	+ 12
68	17 „	24 18.3	150 48.8	5300	978.934	+ 29	+ 22	+ 11

PROVISIONAL RESULTS H. M. S. K XIII (*Continued*).

Nº.	Date 1926	Latitude φ	Longitude λ W. o. Greenw.	Echo depth meters	g_0 sea level	Anom. 1 0.001 cm.	Anom. 2 0.001 cm.	Anom. 3 0.001 cm.
9	13 June	29° 14′	19° 15′	4590	979.278	+ 16	+ 9	— 2
10	14 „	28 39.6	15 53.1	3600	979.210	— 8	— 15	— 26
11	14 „	28 09.3	15 25.2	L. Palm.	979.390	+209	+203	+192
12	22 „	27 11.6	17 51	3710	979.148	+ 40	+ 32	+ 21
13	23 „	26 32.9	21 38.7	4760	979.091	+ 29	+ 22	+ 11
14	24 „	25 44.6	25 19.0	4960?	979.034	+ 30	+ 23	+ 12
15	25 „	25 11.8	28 52.0	?	979.013	+ 49	+ 42	+ 31
16	26 „	25 01	32 30	5900	978.968	+ 15	+ 7	— 4
17	27 „	24 32.2	36 02.7	5930	978.942	+ 22	+ 14	+ 3
18	28 „	24 03.8	39 38.7	5730	978.898	+ 10	+ 2	— 10
19	29 „	23 44.5	43 03.7	3950	978.896	+ 29	+ 21	+ 9
20	30 „	23 20.6	47 02.1	3530	978.887	+ 46	+ 38	+ 26
21	1 July	23 04	50 42.2	4850	978.852	+ 30	+ 22	+ 10
22	2 „	22 45.2	54 34.5	5880	978.804	+ 2	— 6	— 18
23	3 „	22 16.8	58 26.7	5840	978.770	— 2	— 10	— 22
24	4 „	21 34.9	63 21	5650	978.751	+ 23	+ 15	+ 3
25	5 „	20 41.8	65 30.2	5480	978.706	+ 31	+ 23	+ 11
26	5 „	19 30.0	66 50.5	8030	978.284	—321	—329	—341
27	6 „	18 23.7	67 42.3	290	978.549	+ 5	— 3	— 15
28	6 „	16 54	67 42.3	4900	978.460	— 6	— 14	— 26
29	8 „	12 06.38	68 56.10	Curaçao	978.431	+174	+165	+152
30	20 „	12 00.0	69 12.0	1120	978.251	— 2	— 11	— 24
31	21 „	12 55	71 50	1550	978.243	— 45	— 53	— 66
32	23 „	10 25	77 13	3290	978.172	— 26	— 36	— 49
33	23 „	09 55	77 59.5	1450	978.177	— 6	— 15	— 28
34	24 „	09 22.38	79 53.27	Cocosolo	978.250	+ 83	+ 75	+ 62
35	30 „	08 57.53	79 33.93	Balboa	978.249	+ 94	+ 85	+ 72
36	31 „	07 17.3	80 01.8	?	failed			
37	31 „	06 42	80 12	3480	„			
38	1 Aug.	07 01.2	82 37.5	3150	978.168	+ 61	+ 53	+ 40

Generally speaking it must be considered premature to draw already conclusions from the results of the voyage. The observations must first be subjected to the topographic reduction, in which the deviations of the gravity caused by the topographic relief of the earth-surface is taken into account, and to the isostatic reduction giving the deviation caused by the mass deviations in the earth's crust for perfect isostatic equilibrium. Accordingly, this discussion may be postponed to a later occasion.

However, some general remarks and conclusions may already be made, which will not be affected by this reduction. The most important is that both on the Atlantic Ocean and on the Pacific Ocean an excess of gravity occurs throughout large areas. On the Atlantic Ocean the mean over seventeen pure ocean stations, lying between the 15th and the 51st degree of longitude is for instance +0.025 cm., while it can already be communicated now that after isostatic reduction this mean rises to above +0.40 cm. It appears, indeed, from the result of the two ocean stations of the voyage in the K XI, which give: +0.015 and +0.061, that this excess probably extends also to the East as far as Europe.

On the Pacific Ocean the mean of six pure ocean stations between the 141st degree of longitude and Honolulu is +0.030 cm., and of eight similar stations between Honolulu and the 176th degree of longitude East (184th degree of longitude West) is +0.044 cm. By pure ocean stations those stations are understood, for which the gravity is not disturbed by the immediate vicinity of coasts and islands.

It should be stated here that a small excess of gravity might be expected on the oceans, when it is assumed that the earth's crust is in pure isostatic equilibrium, and if the potential planes of the earth would be pure bodies of revolution when we take away the disturbances caused by the irregularities of the crust. For from the distribution of mass present in the earth's crust in the case of perfect isostasy ensues that the potential plane at sea level, i.e. the geoid, lies somewhat higher in the continents than the spheroid of revolution which approximates it as closely as possible, and somewhat lower in the oceans.

For this difference δ of spheroid and geoid the following formula may be easily derived, in which it is supposed that in horizontal direction the ocean has infinite extension.

$$\delta = \frac{H}{g} \triangle'_g$$

in which H is the vertical distance between the centres of gravity of the ocean layer and of the compensating layer of mass under the bottom of the ocean; when the supposition of HAYFORD concerning the isostatic compensation is accepted, H is half the depth of compensation; and in which $\triangle'_g$ is the BOUGEUR reduction caused by the ocean, i.e. the attraction exerted by an infinite layer, having a height equal to the depth of the ocean, and a density equal to the difference of the normal density of the crust and of that of the water of the ocean.

This formula is also valid for the continents and for other topographic deviations which have great horizontal extension. On assumption of an ocean depth of 5000 meters and putting H = 50 km., it gives for the ocean:

$$\delta = 19 \text{ meters.}$$

A closer investigation will afterwards have to decide whether it may be considered permissible, to leave out of consideration the other oceans and continents in the calculation for a definite ocean.

For the time being this amount may, however, serve as basis for a first consideration. It is, indeed, in harmony with the computations carried out in an entirely different way, which have been made in connection with the results of the voyage of the K XIII at the Coast and Geodetic Survey at the order of BOWIE, who first drew the attention to this question. These show that the geoid in the West Atlantic ocean at 32° 47′ 12″ N.Lat. and 71° 00′ 00″ W. Long. lies about 23 meters under the spheroid, and that it lies 6 meters above it in the Appalachian Mountains. Hence together a difference of 29 meters.

From the difference discussed it ensues that the mean gravity reduced to the geoid on the continents must be smaller on the whole than on the oceans, and this by an amount which for the present may be estimated to be at most 0.006 cm. A more thorough computation, which takes the influence of these deviations between geoid and spheroid over the whole earth into account, will have to determine this amount more accurately.

As the normal formulae, with respect to which the anomalies have been computed, have been drawn up from the results of observations on the continent, a positive anomaly is to be expected on the oceans, which we believe to be at most +0.006 cm. The excess being greater, it may be concluded that gravity on the oceans does not perfectly correspond to one of the normal formulae used.

Nor do the results correspond to a term of longitude of the kind as drawn up by HELMERT or HEISKANEN. It is, therefore, also excluded to think of an elliptic cross section of the deeper potential planes with a terrestrial crust which is in itself in isostatic equilibrium. It seems probable therefore that the earth's crust over large areas is not in perfect isostatic equilibrium, or expressed in other words that it presents regional isostatic anomalies 1).

Whether the potential planes would be planes of revolution, when we take away the irregularities of the earth's crust or whether they would present some flattening, cannot be concluded from the investigation of gravity, but there seems to be no reason now to assume a deviation from the circle.

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1) The masses causing these isostatic anomalies, give themselves also rise to a deviation of the geoid, which generally speaking must have the same sign as the anomalies. The deviation of isostatic equilibrium is therefore in general still greater than is apparent from the anomalies. (In translating this note has been rectified according to a more careful consideration of the matter,.

Of course this does not imply that when the gravity results found during this voyage are expanded into series of spherical harmonics, it would not be possible that a term of longitude might appear, though it does not seem probable that, if so, it will be great ; in this case also terms of longitude of higher order than the second will most likely appear. If, however, the condition is made for the normal formula of the gravity that when the anomaly of the gravity reduced to the spheroid of revolution and also reduced isostatically, is zero, this must mean that the earth's crust is in isostatic equilibrium, there is no valid reason now to introduce a term of longitude into this normal formula.

From this it does not ensue, of course, that the two principal moments of inertia of the earth round axes in the equatorial plane, must be equal. The results of this voyage will certainly be an inducement to set on foot a new calculation of this, which has a good chance to approach the true value more closely than was possible with the earlier material, because the stations of this voyage have a much more favourable situation. It is very well possible that then a small difference will appear.

However, as it has just appeared that deviations of the gravity occur over extensive areas, which at present make the impression of not being regularly distributed over the earth's surface, a perfectly accurate value for the difference of the moments of inertia can only be found through further investigations which determine this distribution more closely. A comparatively small number of crossings of the ocean can, however, already do much in this respect. Also the application of the equation of STOKES, which enables us to determine the form of the geoid, when the gravity is known over the whole earth, would then become feasible.

Whether it must really be assumed that the oceans present a positive deviation of the gravity with respect to the isostatic state of equilibrium in the regions mentioned, or whether the normal formula should be changed, and the continents have a negative deviation, whereas the oceans show a normal value, or whether still other possibilities must be assumed as the most probable, may be left undecided here. It is also premature to try and find causes ; this much can only be stated that these equilibrium deviations over such large areas must necessarily be attributed to forces, which are now active over large areas in the earth's crust, or which have been active in recent times ; in which latter case the forces of internal friction are at present responsible for the disturbance of equilibrium.

A further remark that may be made when the results for the Pacific Ocean are considered, is the parallelism between the course of the anomalies and the depth of the sea. This is particularly apparent for the submarine ridges which were passed between Honolulu and Guam.

The first ridge to the west of Honolulu, which is very extensive — its breadth is of the order of some thousand kilometers — does not occur on the charts. In general it may be said that the soundings have revealed a more irregular bottom topography for the two oceans, both for the Atlantic and for the Pacific, than was known according to the charts.

A more detailed discussion of the results of the gravity, for so far as they refer to the other points mentioned, may be postponed to a later occasion for the reasons mentioned before.

The results found above the deeps, which were all crossed at about right angles to the trough direction, in order to get the profiles of gravity as clear as possible, make the impression that all of them are uncompensated, or at least for the greater part uncompensated.

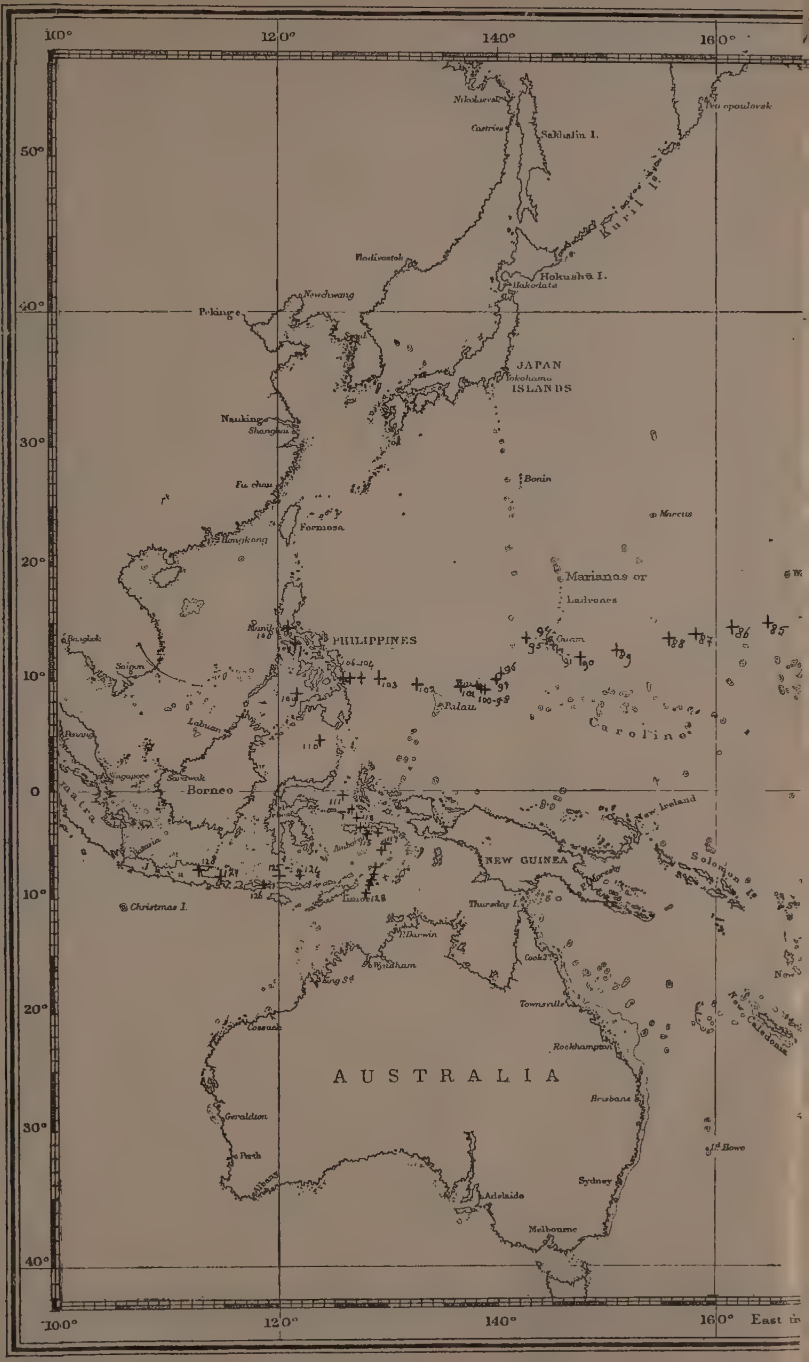
For the Porto-Rico deep there is a very great defect of gravity above the middle of the deep (station N⁰. 26: — 0.329 cm.), and above the borders, where a positive compensation mass lying under the deep would have to make itself felt by a surplus of attraction, there is no question of such a surplus: on the sea-side there is a slight excess, which, however, does not surpass the mean ocean excess; on the island side, in Strait Mona, there is no excess worth mentioning.

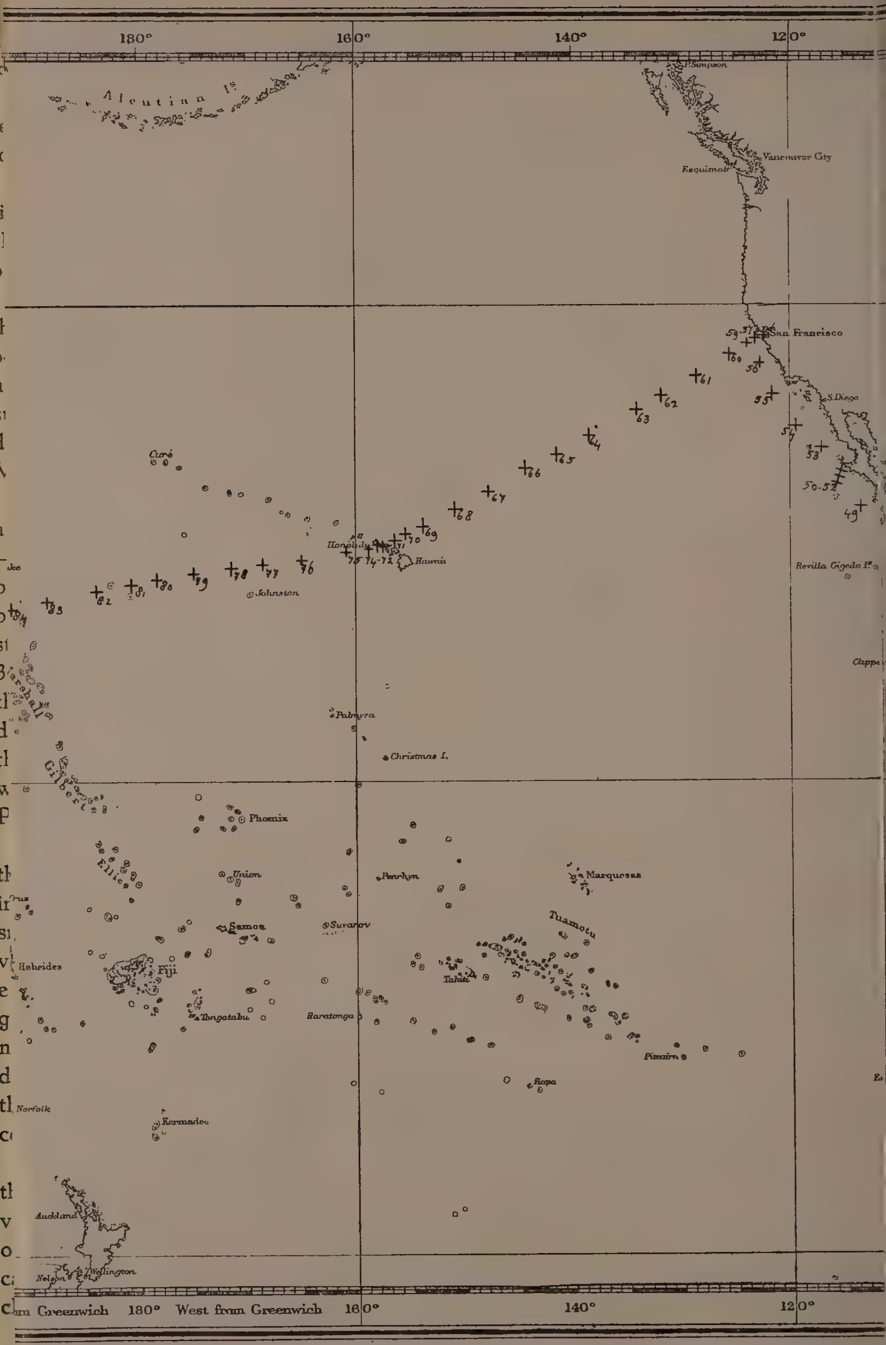
For the Nero deep to the east of Guam there has likewise been found a considerable defect of gravity above the deep (station N⁰. 92: — 0.234 cm.). On the ocean side there is again no excess of any importance present above the border; on the island side, however, there is a very pronounced surplus (+ 0.220 cm.), which also to the west of Guam at station N⁰. 95, where the depth of the sea has already the value of 3610 meters, is pretty great, viz. + 0.062 cm. These results would suggest that the excess is not caused by a positive compensation mass under the deep, but is due to another cause. In my opinion there is here reason to think of a tangential pressure in the earth's crust. Also the excess of gravity, which continues to be found throughout the distance between Guam and the Philippine deep, would point in this direction.

The values for the Yap deep (stations N^os. 97—101), and those for the Philippine deep (stations N^os. 103—106) again give an analogous image: a defect above the middle of the deep, which is here, however, smaller, viz. — 0.146 cm. for the Yap deep and — 0.194 cm. for the very deep Philippine deep, and now on either side of the deeps an excess of gravity (for the Philippine deep on the west side station N⁰. 109 gives e.g. only —0.022 cm. with a depth of the sea of 4870 meters, notwithstanding the coast effect there would have to give a very great defect when there would be perfect isostatic compensation). The course of this excess again does not make the impression that it is caused by compensation masses lying under the deeps.

Remarkable are here too the values for the stations N^os. 107 and 108 in the northern part of the Philippine Islands. Here the excess has almost vanished. Is this perhaps in connection with the fact that on the east side of this part of the Philippine Islands there is no deep present? In this case the view that the excess of gravity in the region of the deep is in close connection with the deep formation, becomes still more probable.

Dr. F. A. VENING MEINESZ: GRAVITY STATIONS DURING THE VOYAGE

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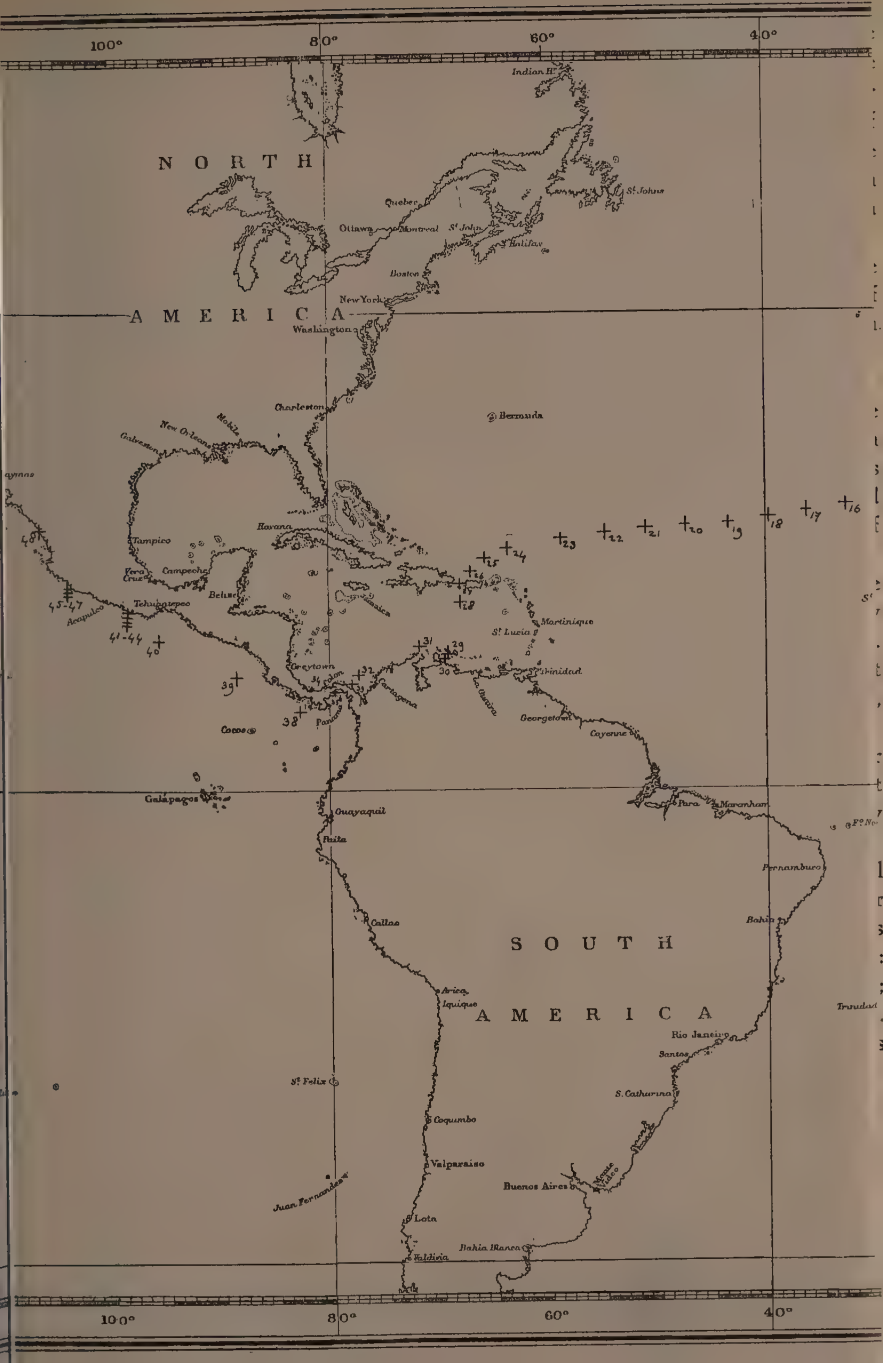
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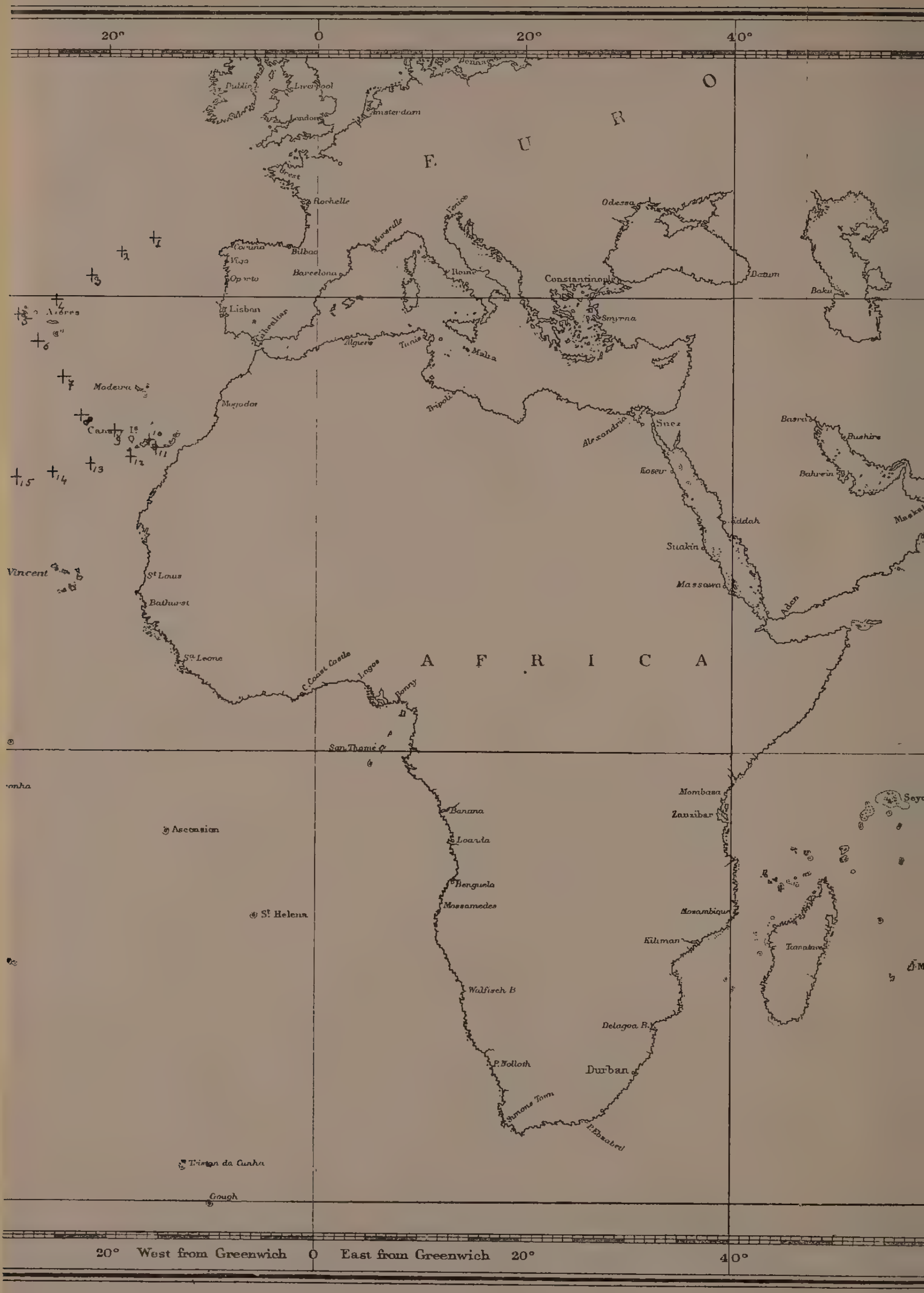
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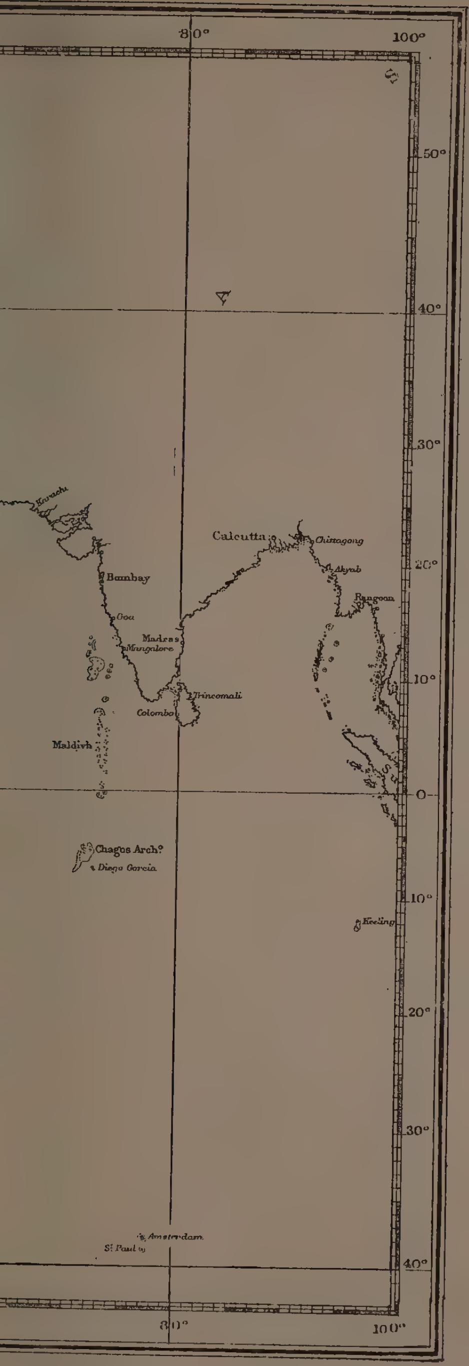
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Between Panama and San Francisco, where an examination of the course of the gravity above the continental border was on the programme, there were made a series of observations above the deep-sea, and further observations at four places above profiles about normal to the lines of equal depth ; two of these profiles have been chosen at spots where there is a deep at the foot of the continental shelf. It may be expected that in particular the series of profile observations will contribute to give us a clearer insight.

It can be stated already now, that the observations obtained above the foot of the continent border, have yielded a considerably greater value of gravity than is compatible with an earth's crust isostatically in equilibrium according to any of the current suppositions.

Only a few brief remarks will be given here about the observations made in the East of the Malay Archipelago. The series of observations have been made above profiles crossing the double arc of Banda Islands ; one was made during the voyage from the North to Banda (Nos. 111—115), and one from Banda via the centre of the arcs southward to the Sahulshelf (Nos. 117—123). Both these series present a remarkable result :

Inside the inner arc a considerable surplus of gravity, but passing to the outside via the arcs a rapid decrease of this surplus, giving place to a very pronounced defect. Over the Sahulshelf itself an excess is found again. The defects of gravity seem remarkable, as the earth's crust in this part of the archipelago might be supposed to be exposed to tangential pressures, which in general might lead us to expect an excess of gravity.

That great anomalies should occur in this part of the earth's crust is for the rest in good harmony with the universally prevailing opinion that movements of the crust took place here in very recent times, and that they may even still be in progress.

In conclusion it may be stated that the results of the voyage in general give a pretty clear image : they make the impression of being more regular than most results of the gravity obtained on the continents. This answers to what might be expected from observations on the ocean on two grounds : First the upper layers, immediately under the observer, are homogeneous ; disturbing masses in the earth's crust are farther distant from the observer. And secondly one of the most capricious causes deforming the earth's crust, the erosion, is eliminated here.

Botany. — *On the so-called traumatotropic curvatures of oat seedlings.* By NANNY TENDELOO. (Communicated by Prof. F. A. F. C. WENT.)

(Communicated at the meeting of June 25, 1927).

According to DOLK (1926) an oat seedling forms a new physiological top about two and a half hours after decapitation. From this top growth-accelerating substances move basipetally. These facts have not as yet been taken into account by those who investigated the curvatures induced by incision of the coleoptile. Another explanation of the so-called traumatotropic curvature may be given in connection with DOLK's findings. In the experiments reported here I studied the relation between curvature and the formation of the new physiological top.

I. *Curvatures of decapitated seedlings.*

Etiolated oat seedlings about 35 to 50 millimeters long, were decapitated with a sharp blade about 5 millimeters below the apex. At the same time the primary leaf was pulled out of its sheath. Immediately after this operation an incision of 180° was made about two millimeters below the cut surface. Within a period of two hours after the decapitation and further operations a few seedlings showed a feeble positive curvature. These curvatures may be accounted for as follows: The decapitation removes the centre of formation of growth-accelerating substances. Only the substances still present in the stump move basipetally. At the intact side nothing hampers the transport of the substance. This is obviously not the case at the wounded side. The humidity of the air being low, the cut surfaces shrivelled a little and a small gap was formed forming a sufficient obstacle for any possible transport, the more so because the gap was quite dry. The insertion of a mica disc was therefore superfluous. Only the growth-accelerating substances present below the cut surface pass downwards in a normal way (see WENT JR., 1926). The positive curvature may therefore be explained by a slight excess of growth promoting substance at the intact side. Four hours after the incision the positive curvature has disappeared and a few plants show negative curvatures. This phenomenon occurred curiously enough only in such seedlings in which the incision was made too deep, passing beyond the centre of the coleoptile. In those cases the upper coleoptile-ring appeared in a slanting position and often broke off entirely from its base. Especially in the latter case the plants showed a strong negative curvature. All plants in which the incision passed exactly through the centre remained quite straight. When the primary leaf had not been removed, its growth pushed off the upper coleoptile ring. In this case again a marked negative curvature ensued.

Table I shows the reaction of a few plants in six consecutive hours.

TABLE I.

Time	straight	+	—
after 2 hours	37	7	0
„ 3 „	39	4	1
„ 4 „	33	1	10
„ 5 „	24	3	17
„ 6 „	21	3	20

From these data it appears that the growth at the wounded flank becomes stronger four hours after decapitation which already indicates the formation of a new physiological top at the cut surface.

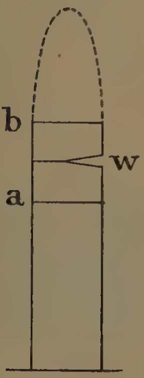

Fig. 1.

Oat seedling. The dotted line represents the top removed by incision; *w* = wound, *b* = coleoptile-ring above the wound, *a* = coleoptile-ring below the wound.

Proof of the formation of this top may be obtained by placing the coleoptile rings *a* (see figure 1) on newly decapitated seedlings. For specific curvatures of the seedlings should be a result of such a regeneration. Three hours after decapitation and wounding the tops were removed by completing the incision already made and placing the rings *a* three to four millimeters high by means of 10 % gelatin, on the new stump. Care must be taken to insure a good contact between ring and stump. The influence of atmospheric- and soil-humidity was very apparent here. If the atmospheric humidity is low the rings will dry out rather rapidly and a dry soil diminishes the reactivity of the plants. The experiments were therefore carried out in a room with constant humidity (87 %) and constant temperature (25° C.).

The plants were controlled every hour and after three hours the side of the original wound became convex. The coleoptile ring therefore produced more growth-substance at the wounded side, caused by formation of a new physiological top at that side. The promptness of the curvature and its rapid development (within three hours) excluded the possibility of another cause. For an imperfect contact between ring and stump should also give rise to a curvature because certain areas of the stump could regenerate a new top independently. The latter process, however should take much more time. The curvatures ensuing within three hours after the application of the ring were only taken into account. When the rings were prepared already two hours after decapitation and wounding, no subsequent rapid curvatures were induced by placing these rings on new stumps. Here apparently the new physiological top has not yet been regenerated and the quantity of growth-accelerating substance remains the same at both flanks. Even five hours after the treatment of the new stumps no curvature could be observed. We will now investigate whether a new physiological top has been formed at the apex of the stump (ring *b* figure 1). In order to test out this fact I placed the ring *b*, three hours after incision on a new stump.

It appeared that a curvature of the stump ensued, showing an acceleration of growth at the flank corresponding to the intact part of the coleoptile from which the ring was obtained. This lends to show that the regeneration of the physiological top had only taken place where a basipetal continuity was assured. Rings *a* and *b* therefore give curvatures in opposite directions which may be demonstrated clearly by placing them on two neighbouring seedlings. When the wound was applied at the right flank the rings *a* caused a convexity at the right flank of the new stump and the rings *b* caused a concavity of the right flank of the new stump (see figure 2).

The experiments also show that the curvatures are not due to the wounding as rings *a* and *b* both contain the same wounded area.

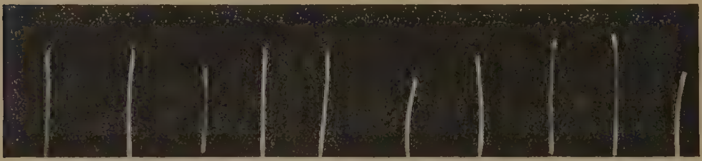

Fig. 2

Decapitation and apical wounding only gives rise to a new physiological top there where a basal continuity is assured.

This appearance of a top is therefore also the cause of the negative curvatures which occur four hours after wounding.

II. *Traumatotropic curvatures of normal, non-decapitated seedlings.*

Etiolated seedlings show a strong positive curvature already one hour after a one sided incision 6 to 7 millimeters below the apex. According to Paál this is due to an interruption in the transport of growth-accelerating substance. Care was taken not to injure the leaf. The insertion of a mica disc was necessary here. If such a disc was not applied, the cut surfaces stuck together and the wound healed more or less.

The cells at the cut surface are about as old as in the case of wounding after decapitation. The curvature of the non-decapitated seedling after wounding is very marked ; a difference in length between the convex and concave flank became visible already after one hour.

These positive curvatures were still more apparent after three hours.

The curvature appears immediately below the wound and proceeds towards the base. After four hours a straightening below the wound becomes visible. The positive curvature in this case remains visible over a much longer period than the curvature after decapitation. Table II shows the amount of curved and straight plants at various times after the wound had been applied.

TABLE II.

Time	straight	+	—
after 1 hour	4	23	0
„ 2 hours	2	25	0
„ 3 „	3	24	0
„ 4 „	8	19	0
„ 5 „	10	14	3

After 20—24 hours several plants are entirely straight especially the small ones (about 20 millimeters). It is doubtful whether these straightened plants are comparable to normal, unwounded plants. For the straightening and even convexity of the wounded flank may be due to the formation of new growth-promoting substances below the incision. What happens at this surface three to four hours after incision ? This question has been studied by the methods mentioned above.

Three hours after wounding the ring *a* (Fig. 1) below the incision was placed by means of 10 % gelatin on a new stump. In almost all stumps curvatures appeared, the convex flank of the stump situated directly below the originally cut surface. Figure 3 depicts these plants. Application of rings *b* on new stumps did not cause any curvature. No increase of growth substance has therefore taken place above the wound. No accumulation of this substance above the wounded surface could be demonstrated. No physiological top is regenerated above the incision. The plants which were quite straight after 24 hours are therefore not comparable to normal intact seedlings.

It appears from the above that it is desirable to observe the plants in short intervals after the wound has been inflicted. For the regeneration of the new physiological top may cause a transition from positive to negative curvatures.

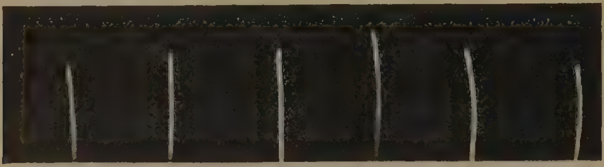

Fig. 3.

Curvatures which ensue after unilateral wounding have two causes:

a. The curvatures which appear within the first three hours after incision are caused by the interruption in the transport of growth-promoting substances.

b. The curvatures which appear three hours (and later) after incision, are caused by the formation of a new physiological top at the cut surface. The observed transition from positive to negative curvatures is due to the antagonistic action of factors *a* and *b*.

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A. PAAL: 1914. Ueber phototropische Reizleitung. Berichte d. deutsch. bot. Gesellschaft, Band **32**.

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Utrecht, June 1927. *Botanic Laboratory.*

Chemistry. — *Osmosis of ternary liquids. General considerations.* I.
By F. A. H. SCHREINEMAKERS.

(Communicated at the meeting of September 24, 1927).

In previous communications[1]) which will be continued, I have discussed some equilibria, which may occur in systems, in which one or more semipermeable membranes are present. As, however, membranes, which let some of the dissolved substance pass, and others not, are difficult to make in a serviceable state[2]), now I have undertaken in collaboration with Mr. B. C. VAN BALEN WALTER an experimental investigation on the progress of the osmosis, if membranes are used, which are permeable for all substances. In order to facilitate the discussion of the results, which are sometimes very remarkable at first sight and in order to give some survey of that which is to be expected in this sphere, it is necessary to begin with some general considerations. As our experimental investigations, which will be published afterwards, relate to liquids, which consist of three substances, I shall limit myself in my considerations to these liquids.

1. *The theoretical osmosis-path.*

We take a liquid L consisting of three substances, which we shall call X, Y and W; although it is not of the least importance for the further, considerations, we shall say that W represents water; we may imagine arbitrarily to be the substances X and Y e.g. $NaCl$ and Na_2CO_3 or sugar and ureum, etc.

We represent the composition of this liquid L by:

$$x \text{ quant. of } X + y \text{ quant. of } Y + (1-x-y) \text{ quant. of } W$$

in which we mean by "quantities" either quantities of weight or mol-quantities. Of course in general it is indifferent in which of those "quantities" we express the composition; yet we shall see afterwards that in some cases we have to pay attention to it.

We are able to represent the composition of a liquid in the well known way graphically in a concentration-diagram; for this purpose we take a triangle, the anglepoints of which represent the substances X, Y and W; we choose this triangle equilateral or rectangular equilateral. In fig. 3 the sides WX and WY of a similar triangle are partly drawn.

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[1]) F. A. H. SCHREINEMAKERS. These Proceedings, 27, 701 and the 23 following communications.

[2]) In his book "The osmotic pressure of Aqueous Solutions" K. N. MORSE describes a method of making membranes, which are permeable fot water only. Although those membranes are quite fit for that purpose, we cannot use them, because they transmit the water too slowly.

Now we shall represent the osmotic system, which arises, if we bring in osmotic contact bij means of a membrane the liquids L_1 and L'_1 by:

$$L_1 \mid L'_1 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

If we wish to express also that there are n_1 quantities of the liquid L_1 and n'_1 quantities of the liquid L'_1, then we represent it by:

$$n_1 \times L_1 \mid n'_1 \times L'_1 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

If the membrane is transmitting the three substances, then this system cannot be in equilibrium, unless both liquids have the same composition.[1]) Consequently the substances X, Y and W must diffuse through the membrane in some direction and this osmosis will continue as long as the liquids on both sides of the membrane get the same composition; we represent this final-liquid by L_e.

If we represent the composition of L_1 by:

$$x_1 \text{ quant. of } X + y_1 \text{ quant. of } Y + (1 - x_1 - y_1) \text{ quant. of } W \quad (2^a)$$

and that of the liquid L'_1 by

$$x'_1 \text{ quant. of } X + y'_1 \text{ quant. of } Y + (1 - x'_1 - y'_1) \text{ quant. of } W \quad (2^b)$$

and that of the final-liquid by:

$$x_e \text{ quant. of } X + y_e \text{ quant. of } Y + (1 - x_e - y_e) \text{ quant. of } W$$

then we find easily:

$$x_e = \frac{n_1 x_1 + n'_1 x'_1}{n_1 + n'_1} \qquad y_e = \frac{n_1 y_1 + n'_1 y'_1}{n_1 + n'_1} \quad . \; . \; . \; . \quad (3)$$

Therefore the composition of the final-liquid L_e does not only depend on the compositions of the liquids L_1 and L'_1 but also on the quantities n_1 and n'_1 which are present in system (2).

We represent the liquids L_1 and L'_1 of system (2) in fig. 1 by the points 1 and 1′. In order to save room the anglepoints of the sides of the concentration-triangle WXY are omitted in this figure. We represent the final liquid L_e by a point e which is of course somewhere on the straight line 1 . 1′. The composition of the final-liquid e and therefore also the position of the point e depends on the ratio $n_1 : n'_1$ consequently on the ratio of the quantities of the liquids on the left and on the right side of the membrane. The point e divides the line $1 . 1'$ into two parts $e . 1$ and $e . 1'$ the lengths of which are defined by:

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[1]) If the three substances form a system, in which dimixtion into two liquids can occur, then system (2) can also be in equilibrium when L_1 and L'_1 are different. Then, however, L_1 and L'_1 represent two liquids, which can exist next to one another if we take away the membrane.

$$e . 1 : e . 1' = n'_1 : n_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In order to pass into the final-liquid L_e the liquid L_1 will go along a path, which starts from point 1 and finishes in point e; this path which we shall call the „osmosis-path of the liquid L_1" is represented in fig. 1 by curve $1 . 2 . 3 \ldots . e$; the arrows indicate the direction in which the liquid travels along this path during the osmosis. The same is valid for the liquid L'_1; its osmosis-path is indicated by curve $1' . 2' . 3' \ldots e$. Those two osmosis-paths of the liquids form together the osmosis-path of system (2).

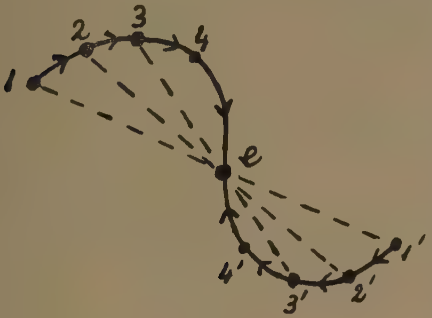

Fig. 1.

After some time the liquids L_1 and L'_1 of this system will have changed their composition; e.g. the left has arrived at point 2 and the right in point $2'$; we now call those liquids L_2 and L'_2. As the quantities of the liquids on both sides of the membrane change also during the osmosis, we represent them now by n_2 and n'_2. Consequently system (2) has passed into the system:

$$n_2 \times L_2 \mid n'_2 \times L'_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

As the total quantity of both liquids does not change during the osmosis, must be $n_2 + n'_2 = n_1 + n'_1$. As also the total composition of the whole system remains constant, the line $2 . 2'$ must also go through the point e. Instead of (4) we now have:

$$e . 2 : e . 2' = n'_2 : n_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Some time afterwards the liquids have arrived in the points 3 and $3'$; we then have the osmotic system:

$$n_3 \times L_3 \mid n'_3 \times L'_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

For this is valid:

$$n_3 + n'_3 = n_2 + n'_2 = n_1 + n'_1.$$

Of course the line $3 . 3'$ goes also through the point e; we now have:

$$e . 3 : e . 3' = n'_3 : n_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Of course the same thing holds good also when the liquids are in other points of their paths, e.g. in 4 and $4'$; of course the line $4 . 4'$ goes also through point e.

We shall call liquids which are present at a same moment on both sides of the membrane, as e.g. L_1 and L'_1 or L_2 and L'_2 etc. „conjugated liquids". The lines which unite two conjugated points, as e.g. $1 . 1'$ or $2 . 2'$ etc. are called „conjugation-lines". From the above now follows:

the conjugation-lines of an osmosis-path all go through the point which represents the composition of the final-liquid; the ratio of the parts into which this point divides the conjugation-lines, is equal at any moment to the ratio of the quantities of the liquids which are present at that moment on both sides of the membrane.

On the reverse: every straight line, going through the point e, which intersects the osmosis-path is a conjugation-line; as the points of intersection represent two liquids which will be present at a definite moment on both sides of the membrane.

If we draw tangents to the osmosis-path in two conjugated points e.g. in 1 and $1'$ or 2 and $2'$ then we call them "conjugated tangents"; if in the one path we draw a chord e.g. $1 . 2$ or $2 . 3$ etc. and in the other path the corresponding chord e.g. $1' . 2'$ or $2' . 3'$ etc., then we call them "conjugated chords". Later we shall see how we can make use of the conjugated tangents and chords in order to deduce the directions and the ratios in wich the different substances diffuse through the membrane at definite moments.

If in the osmotic system (2) we take two definite liquids L_1 and L'_1 then the osmosis-path starts in fig. 1 from the points 1 en $1'$; the form of the path, however, still depends on many factors; some of them are:

a. the quantities n_1 and n'_1 of the liquids on both sides of the membrane;

b. the membrane through which the osmosis takes place;

c. the temperature and pressure.

The influence of that mentioned sub *a* is evident; above we have indeed seen already that the position of point e depends on the ratio $n_1 : n'_1$ consequently on the ratio of the quantities of the liquids L_1 and L'_1.

If we take the same quantity of liquid on both sides of the membrane, so that $n_1 = n'_1$ then as becomes apparent from (4) also $e . 1 = e . 1'$; then the point e is situated, as in fig. 2, in the middle of the line $1 . 1'$; then the osmosis-path can have a form as curve $1\,e\,1'$.

If, however, we take more of the liquid L_1 so that $n_1 > n'_1$, then follows from (4) that $e . 1' > e . 1$; so the final-point e is situated nearer to point 1; it is represented in fig. 2 by point q. Consequently the osmosis-path must now have also another form; it is represented by $1\,q\,1'$.

If we take more of the liquid L'_1 so that $n_1 < n'_1$ then follows that the final-point e must be situated nearer to point $1'$; it is represented in fig. 2 by point r; the osmosis-path now can have a form as curve $1\,r\,1'$.

We shall afterwards discuss some examples of such paths.

Of course for each of these paths is true that the conjugation-lines go through one point; these of path $1\,q\,1'$ all go through point q; these of path $1\,e\,1'$ through point e and these of path $1\,r\,1'$ through point r.

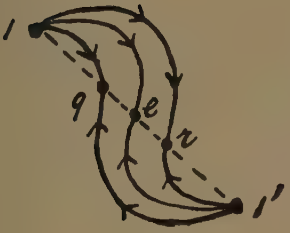

Fig. 2.

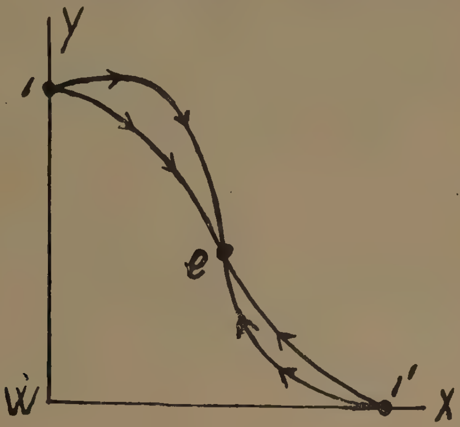

Fig. 3.

We are also able to show that those paths must touch one another in their starting points 1 and 1'.

The rate at which the different substances diffuse through the membrane and consequently also the ratio in which this takes place at every moment, is also dependent on the nature of the membrane; therefore, this also defines the form of the osmosis-path.

Let us take as example an osmotic system

$$L_1 \mid L_1' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

in which L_1 only contains the substances W and Y and the liquid L'_1 only the substances W and X. Then the liquid L_1 is represented in fig. 3 (in which the sides WX and WY of the concentration-triangle are only partly drawn) by a point 1 on the side WY; the liquid 1' is indicated by a point 1' on the side WX. As soon as the diffusion of the substances through the membrane has begun, each of the liquids contains three substances; they are represented by points within the triangle.

It now depends on the kind of membrane along which paths the liquids will travel; in fig. 3 two osmosis-paths are drawn; we have defined them in the system:

$$W = \textit{water} \qquad X = Na_2S_2O_6 \qquad Y = BaS_2O_6.$$

Consequently the liquid L_1 contained: water + BaS_2O_6 and the liquid L'_1: water + $Na_2S_2O_6$.

2. *The experimental osmosis-path.*

We shall call the osmosis-path, discussed above, the "theoretical" osmosis-path of a system. In order to define this path experimentally, we have to take away, however, now and then a small quantity of the two liquids in order to analyse them. We shall show that the system now no longer travels along the theoretical path, but along a path, more or less modified, which we shall call the "experimental".

We take two liquids L_1 and L'_1 the composition of which is known; if we bring n_1 quantities of L_1 on the one side and n'_1 quantities L'_1 on the other side of the membrane, then we have the osmotic system:

$$n_1 \times L_1 \mid n'_1 \times L'_1 \quad \ldots \ldots \ldots \quad (10)$$

We represent the two liquids L_1 and L'_1 and the final-liquid L_e in fig. 1 by the points 1, 1′ and e; as yet we eliminate the osmosis-path and its conjugation-lines from the figure.

After a certain time system (10) has passed into the still unknown system:

$$n_2 \times L_2 \mid n'_2 \times L'_2 \quad \ldots \ldots \ldots \quad (11)$$

In order to learn the composition of the liquids we take away a small quantity of the liquids on both sides of the membrane and we determine their compositions; we represent it in fig. 1 by the points 2 and 2′; of course the conjugation-line passes through the point e. During the osmosis the liquids have now gone along the theoretical path 1.2 and 1′. 2′, of which, however, only the points 1, 2, 1′ and 2′ are known.

As we have taken away a little of the liquids L_2 and L'_2, we have no longer the system (11) which has arisen from (10) but a new system:

$$(n_2 - q_2) \times L_2 \mid (n'_2 - q'_2) \times L'_2 \quad \ldots \ldots \quad (12)$$

in which q_2 and q'_2 are the quantities taken away; consequently the systems (11) and (12) consist of the same liquids, but the quantities of those liquids are different.

Above (comp. fig. 2) we have seen already that the osmosis-path of a system depends not only on the composition of the two liquids, but also on the quantities which are present on both sides of the membrane. System (11) which, however, we have no more, would therefore travel along a path differing slightly from system (12) which we do have.

When system (12) has passed after some time into the system:

$$n_3 \times L_3 \mid n'_3 \times L'_3 \quad \ldots \ldots \ldots \quad (13)$$

then we are able to ascertain again the composition of both the liquids; we imagine them to be represented by the points 3 and 3′. The liquids have travelled now along the paths 2.3 and 2′. 3′; they are the theoretical paths of system (12) and, therefore, not those of the original system. Consequently the curves 1 . 2 and 2 . 3 are parts of several theoretical paths; so their prolongations do not coincide. It follows, however, from our previous considerations that both paths touch one another in point 2, so that in this point not a discontinuous transition but a continuous transition takes place.

In the same way it follows that the theoretical path 3 . 4 is again somewhat different from the theoretical path 2 . 3; etc.

The difference between the two paths 1 . 2 and 2 . 3 in the point 2 depends on the difference, between the two ratios:

$$n_2 : n'_2 \text{ and } (n_2 - q_2) : (n'_2 - q'_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The less those ratios differ, the less also the paths 1 . 2 and 2 . 3 differ. If on both sides of the membrane we take away quantities q_2 and q'_2 which are small with respect to the present quantities n_2 and n'_2 then the both ratios (14) are equal approximately and the theoretical path 2 . 3 is also equal approximately to the theoretical path 1 . 2.

If on both sides of the membrane we take away an equal part of the liquids, then we have:

$$q_2 : q'_2 = n_2 : n'_2$$

both ratios (14) have now become equal and the paths, 1 . 2 and 2 . 3 now form the same theoretical path. Consequently we may say:

the experimental osmosis-path consists of a succession of parts of theoretical paths, which pass into one another in the points, experimentally defined; this transition takes place continuously. The less their ratio is changed when a small quantity of the liquids is taken away, the less those theoretical paths differ from each other.

Above we have seen that the conjugation-lines of a theoretical path intersect one another all at the same point, consequently all conjugation-lines of points of the path 1 . 2 with points of the path 1′. 2′ go through point e. All conjugation-lines of points of the path 2 . 3 with points of the path 2′ . 3′, therefore, go also through a same point; this, however, is not the point e but another point, not indicated in the figure, on the line 2 . 2′ which we shall call h.

Previously we have seen that the position of point e on the line 2 . 2′ is defined by:

$$e . 2 : e . 2' = n'_2 : n_2$$

The position of point h on the line 2 . 2′ is defined by:

$$h . 2 : h . 2' = (n'_2 - q'_2) : (n_2 - q_2)$$

This shows that the ratios, which define the difference in the position of the points e and h are the same as the ratios (14), which define the difference of the two theoretical paths 1 . 2 and 2 . 3. Just as the theoretical paths themselves, consequently also the points e and h will coincide the more, the less we change their ratio on taking away a little of the liquids.

It appears from the previous considerations that it is possible to define experimental paths, which practically coincide with the theoretical. For this reason we shall first start from the theoretical paths in our following considerations.

(To be continued).

Leiden, Lab. of inorganic chemistry.

Genetics. — *Heredity of mental faculties.* By J. F. VAN BEMMELEN.

(Communicated at the meeting of December 18, 1926) [1].

In his interesting book: "Mental and moral Heredity in Royalty", WOODS makes an attempt to estimate the proportionate influence of the three great causes that are believed to govern mental and moral development as well as physical characteristics s.c. heredity, environment and free-will.

As might be expected, the latter cause being of an "ultranatural character" proved to be unfit for scientific investigation, so that only the two former causes remained to be outweighed against each other. Of these, hereditary predisposition was found to be far the most powerful causation, in fact to be the primary moving agency, the effects of which were only modified or counteracted in a superficial way by outward circumstances.

In many cases the evidence adduced by the investigation of the ancestors of his objects proved so complete and was so convincing, that WOODS repeatedly could point out how the peculiarities in the offspring of a certain royal couple were in perfect harmony with what might have been predicted from the character of the parents. In two cases: that of the great CONDÉ and that of TURENNE, he even went so far as to state that they appear in the very place in the pedigree where they might have been expected (compare pag. 491, when he says: "the greatest of the grandchildren (of WILLIAM THE SILENT), TURENNE, occurs where he would most probably fall").

Notwithstanding this apparently brilliant result of his investigations, it seems to me that WOODS' methods are open to severe criticism. He applies a system of estimation or valuation, providing each of his study-objects with two gradings, one for intellect, the other for morals. His scale of gradation runs from 1 to 10, and his denotation may be best understood by an example: CATHARINA OF RUSSIA is labelled 10 for intellect, 1 for morals, and moreover is marked with a black cross: indicating depravedness, licentiousness, cruelty and other detestable aberrations of human nature.

Now from a biological point of view — the only one that can be admitted in matters of heredity, as WOODS himself fully acknowledges, when he says: "History is really but a branch of biology" — this method is utterly unscientific. For mental qualities should be defined and measured according to their own special character exactly like corporal properties, that is to say they are to be studied with no references whatever to their social merits or demerits, or to their appreciation by contemporaries or posterity. In biology, a moral standard is entirely out of place. Indeed the effect of a certain inborn quality or endowment, both on the welfare of the organism possessed by it and on that of its surroundings, can never be utilized as a standard for determining its biological nature or for measuring its grade.

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[1] Dutch publications in "Verslag van de gewone vergadering" of September 1927, Section of Science, Royal Acad. Amsterdam, Vol. XXXVI, p. 787.

All phenomena of heredity depend on the presence or absence of innate factors, which can only be defined in accordance with their own specific constitution, not with the desirability or the danger of their influence. By a slight modification of circumstances (internal as well as external) one and the same hereditary property, occurring in different members of the same family, may assume diametrically opposed aspects: it may appear as a virtue in one of them, as a vice in a second, while in a third it may remain perfectly hidden, and yet all three are equally likely to transfer it to their offspring.

This consideration moreover shows us the misleading effects of any attempt towards gradation or valuation of mental qualities. For this reason no mental endowments any more than somatic ones, should ever be characterized or classified according to the variable impression made by their different manifestations on the minds of contemplators. To give an example: it cannot reasonably be doubted that JOHAN SEBASTIAN BACH belonged to a family, which in several succeeding generations produced an extraordinary number of musically endowed persons. It is equally well-known that all of his fourteen children were more or less gifted in this direction. But the words more or less make it evident, that there existed certain gradations in the amount of musicality of these different children, and indeed it would be difficult to understand how it could have been otherwise. For musicality can scarcely depend upon a single hereditary factor, or even on only a few and intimately correlated ones. This must form a serious obstacle against the application of Mendelian principles to such intricate cases of mental heredity. The same objection, according to my view, may be raised against almost every case of apparent segregation of these psychic qualities in human descent. The chance that a special giftedness for certain qualities should remain concealed for a certain period, and therefore escape notice, is so great, that it must exercise a very serious influence on the results of statistical investigations.

I myself have witnessed the fact, that a medical man with a large rural practice, nearly fourty years of age, quite accidentally became aware of his being gifted with a fine talent for the plastic arts. He had bought a box of plasticine for a christmas present to his children, and in showing them the use of this material, he modelled the pet goat of the family in such a masterly way, that it was at once clear that he was greatly gifted as an artist. Had he ended his days without making this discovery, and had the hidden talent manifested itself in one or more of his offspring, this phenomenon might have been considered as a well-authenticated case of spontaneous variation. The misleading effect of this source for mistakes becomes clearly apparent in one of the main conclusions of WOODS, where he contends that WILLIAM OF ORANGE, in contradistinction to his numerous offspring female as well as male, was a novum. His own words are, p. 87: "The father and mother of WILLIAM THE SILENT were both excellent characters, and the son doubtless inherited many good qualities from them, but neither the parents or other ancestors were in any way remarkable, so WILLIAM THE SILENT must be considered a new variation (mutation or sport). Although he himself cannot be taken as an instance of heredity, all his descendants can".

The utter improbability of this conclusion need not be pointed out. Every Dutchman moreover who knows anything of his national history, is at present in a position to show that WOODS, in this particular instance, is totally wrong. Exactly as in the case of GOETHE, the genius of the son can almost completely be explained as a higher differentiation of the great qualities of the mother, the difference between parent and child not arising from constitutional contrast but being mainly one of fame. It is only thanks to the lucky and quite accidental preservation of the correspondence of JULIANA VAN STOLBERG with her children (recently published) that to-day we have an insight into the admirable and heroic character of this wonderful mother of WILLIAM THE SILENT, and know her to have been endowed with all the qualities of brain and heart, that brought her son immortal glory throughout the world.

In the choice of the mental qualities, that can serve as tests in tracing the course of heredity, we should therefore take no notice whatever of public appreciation and prevailing opinions. We should by all means try to prevent ourselves from being influenced by moral conviction and traditions, and never agree with the popular explanation of a certain phenomenon, merely on the ground that it is generally believed to be right. Popular explanations and convictions are nearly always erroneous, just as in the case of bodily qualities and conditions. To cite an example: when in the succeeding generations of a family a majority of its masculine members choose the same career, say the military profession, this is commonly attributed to the influence of education and tradition or even moral pressure on the side of the parents. That it might with much more probability depend on a hereditary predisposition often manifesting itself quite apart and unconnected with or even in plain conflict with the outward circumstances, is far from being generally recognized. Yet I am convinced that the latter view is the right one, and that it will be susceptible of proof in every case, provided two conditions to be fulfilled. The first condition is that the quality which is chosen as an object of investigation, must possess a well-marked special character, enabling it to be distinguished from others near akin to it, under whatever outer appearance the said property manifests itself.

The second condition is that all these different manifestations must be considered of equal value, without admitting even the slightest influence on our judgment from the side of moral conviction, social tradition or general approval, and what is even more important, without enquiring if the quality under discussion possesses a normal or abnormal, a physiological or a pathological character.

The merits and the difficulties of a system of investigation, which keeps to the above-mentioned conditions, can best be illustrated in the case of hereditary talents. If a series of generations of artists occurs showing a talent of the same nature, nobody doubts that this phenomenon can only be explained by the presence of a hereditary tendency for the art in question. The greatest painter or musician will never be able to raise his children to his level, if they are not specially gifted by birth in the same direction. On the other hand, we shall never be able to get a complete insight into the course of heredity, if we do not pay attention to all the descendants of a certain gifted ancestor, down to the least gifted among them. But this unavoidable necessity places us for the difficulty, that by extending our researches to the whole of the offspring, without any exception, we are encountered with a host of descendants that show the mental property in a mediocre degree not exceeding or not even reaching its average level among the majority of mankind. Most people are musical to a slight extent, and are able to draw a rough sketch of a simple object. But even so this method may yield reliable results, if we succeed in specialising the character of a certain hereditary feature, by restricting our attention on certain details in its constitution.

Of this kind of research, I wish to give two instances, both taken from my own family. In biological matters, the family- and christian names of course are without any significance whatever, and had better be altogether omitted. When I mention the names of the members of my pedigree in a table at the end of this paper, I only do so to facilitate later controle, and to enable future generations to extend the list.

The pedigree itself forms a so-called Parentela, which means a survey of all the descendants of a certain pair. The members of the parentela have been numbered in a continuous series, in connection with their places in the pedigree, but without paying attention to the date of their birth.

The quality I wish to discuss I should like to define as: "Tendency and fitness to administer instruction to people". From a genetic point of view, the tendency towards didactic occupation seems to me of still greater importance than the fitness for it. For this latter may be handicapped or checked by numerous circumstances, inner as well as external, and thereby sink to a level low enough to make its presence doubtful. On the other hand, the criterium which should be applied in deciding whether the quality under discussion is present or not in a certain person, should never be restricted to the question, if he or she had really been occupied in practical instruction or not. Among women especially there must be a very large percentage of cases where the natural tendency towards didactic occupation does not find an outlet for lack of opportunity, thus escaping notice.

The number of potential errors, resulting from this, may be reduced if we take into account, that the inborn tendency towards instruction may manifest itself in various other ways than the actual giving of lessons or other forms of practical teaching. Writing didactic works, or giving active support to public institutions for the promotion or the distribution of knowledge, or — the reverse to some extent and yet the same — the private instruction and education of one's own children, are just as convincing proofs of the presence of the mental tendency in discussion.

Nor need the idea "instruction" be restricted to what is usually considered as the essential and characteristic object of it : imparting the knowledge of science and letters. It may as well be extended to the occupations of the clergyman explaining the doctrines of religion, the musician, the draughtsman and even the artisan and skilled labourer, teaching their pupils the elements of their art or handicraft. But especially in these latter departments the ability for personal achievements must be strictly separated from the fitness and still more from the desire to impart their knowledge or art to others.

In the course of my investigations, it became clear to me, that I might go even further, and include in it all tendencies to offer advice and to act as a mentor to one's fellow-creatures. Fitness for directorship, for commanding (in civil life as well as in military service) for administering legal, financial or commercial advice, may all be considered as manifestations of the same mental disposition.

Yet I thought it advisable to keep the latter categories apart from the capacity for instruction proper. For this reason I have compiled two columns in addition to those tabulating the fitness for instruction in its restricted sense, viz. one column for "Fitness for Directorship", and another for "Influencing public opinion".

As for the grounds on which I felt justified to assign the positive mark in one or more of the five different categories of my list, to special members of the parentela, it would evidently be impossible and quite out of place to discuss them here for every case in particular.

In the majority of instances, especially among the male members, the public office of the person in question, his publications or his social achievements, made the decision self-evident and reliable. Where these direct proofs failed, my personal acquaintance with many of the younger members of the parentela often helped me in the choice. Yet I am fully conscious that I must have committed errors, but I am convinced that these more often consist in omissions, caused by ignorance of the real nature of the persons in question, than in false attribution of presumed qualities. As for partiality in my estimate, I can only say that I have done my utmost to keep my judgment as unbiassed as possible.

Therefore I will now only set forth some general conclusions and special remarks, which seem to me deducible from the parentela as a whole.

In the first place it tells us in a very conclusive way, that parental influence and family-tradition only play a very subordinate part in the choice of the career for the offspring. Mark e.g. the three sons (Nrs. 45, 47, 48) of N⁰. 26, who lost their father very early, and were exclusively educated by their mother, who belonged to a patrician merchant family, in which aptitude for instruction had not become apparent up to that time, and a didactic career was in no way predilected.

Or in the other branch of the family tree the three daughters (Nrs. 95, 96, 97) of N⁰. 59, who returned to the practice of teaching, though the vocation of following this career had long since disappeared, if ever they and their ancestors had consciously been aware that it existed in the family.

Similarly the son (N⁰. 122) and the eldest daughter (N⁰. 123) of N⁰. 79 both took up giving lessons at a time when they themselves were still students. They did that of their own free will, with no animation whatever from the side of their parents, who indeed were surprised at their success in teaching.

Among the children of N⁰. 84, the younger brother of N⁰. 79, the same phenomenon showed itself in a somewhat modified way. The eldest son (N⁰. 128) and daughter (N⁰. 129) first went in for drawing and painting, but though they proved to be gifted as artists, and even produced some remarkable works in that way, they both of their own free will abandoned the idea of an artistic career and took up elementary teaching, for which they both turned out to be especially fit.

But the most convincing case according to my view is that of N⁰. 92, a man crippled in his youth — I presume by the same hereditary disposition that caused the death of so many of his uncles and aunts at an early age — and who remained unmarried.

He started coaching his fellow-students while he himself had not yet finished his legal studies, and continued this successful occupation till far advanced in life, besides teaching his favourite subjects at several high-schools. Even when his health began to fail, and all his friends advised him to retire from a task which he evidently was no longer equal to, he could not decide to yield to the unevitable, and kept to his duties until four days before his death.

In a third branch of the parentela, formed by the issue of N⁰. 22, a very interesting case occurs of what might be called reawakening of a latent faculty. This N⁰. 22 had five sons (N⁰. 35 to 39), one of whom only (N⁰. 35) may be counted among the persons directly interested in didactic work, as he became a clergyman. Yet the rest of them, in their manifold occupations, all showed a tendency — and greater or lesser ability — for giving direction to other men.

The next generation (N⁰. 66 to 77) besides three women, counted nine men, of whom no less than eight followed a military career, while only one became a clergyman like his father. Without any doubt this sudden predominance of the military element may be mainly attributed to straightened financial circumstances of the parents. Moreover, as the occupations of a military commander mainly consist of directing others, these eight men all fit in very satisfactorily with our scheme. But in the following filiation (N⁰. 104—121), counting eighteen members, eight males and ten females, the number of military officers decreased to two. Now one of these N⁰. 114, in the prime of his manly age, was forced to retire from military service by an accident. Though already father of a family, he turned to academical studies, and in a remarkably short time passed all his examinations for the medical degree, and intended to become a practitioner. That however his real vocation lay in the direction of didactic work, had already become so clearly apparent, that he was appointed a University professor only a few years after taking his doctor's degree. Moreover it was proved by the fact, that he became the author of several learned treatises, and a great number of popular articles.

As already mentioned, I wish, in connection with my subject, to avoid even the appearance of an application of Mendelian principles about segregation of features. Therefore I have given up my original plan of arranging all the members of the parentela in the well-known schematic table with black and white sex-marks, indicating the presence or absence of the property in discussion. Had I done so however, a glance at the table would have taught the reader that though the faculty in question had in fact been apparent in an unbroken succession of five generations, this had only occurred in one single branch of the parentela. This particular line may be summarised as follows :

TABLE A.

Parentation No. 1 ♂ (?) Decorator of Delft-pottery.

I Filiation No. 2 ♂ (?) „ „ „

II Filiation No. 14 ♂ Headmaster of a boardingschool, author of many treatises for elementary instruction.

III Filiation No. 26 ♂ Doctor of letters, headmaster at a Grammar-school (Dutch gymnasium) (N.B. died young).

IV Filiation No. 47 ♂ Doctor of chemistry, headmaster of a secondary school, University professor, author.

V Filiation

- No. 79 ♂ Doctor of Zoology Teacher of sec. school. University professor, Author. (talent f. drawing).
- No. 80 ♀ Student of mathematics Elementary teaching
- No. 81 ♀ Authoress
- No. 84 ♂ Doctor of physics. Director Meteorological Observatory. University lecturer. Author. (talent f. drawing).

VI Filiation

- No. 122 ♂ Doctor of law secondary teacher. Author, (talent for drawing).
- No. 123 ♀ Student of music. elementary teaching
- No. 128 ♂ Elementary teaching Author. (talent for drawing and painting).
- No. 129 ♀ Elementary teaching. (talent f. drawing and painting).
- No. 130 ♂ Doctor of geology assistant-lecturer Technical High-school. Author.

It should be expressly remarked that neither this table, nor the other two given below, can be considered as Mendelian schemata in the strict sense of the word, as they nowhere contain both the parents or their complete offspring, but only those members of the pedigree that either showed the property under discussion themselves, or transmitted it to their descendants.

When dealing with simple somatic hereditary features, such an uninterrupted recurrence of the same quality would undoubtedly be considered as a proof that the feature in question possessed the character of a dominant. But in our parentela the mental disposition under discussion appears likewise in several other lines of descent, only here it does not show itself in a continuous sequence, but with intervals of one or more filiations, in which the faculty was apparently absent. This may be seen from the following two examples.

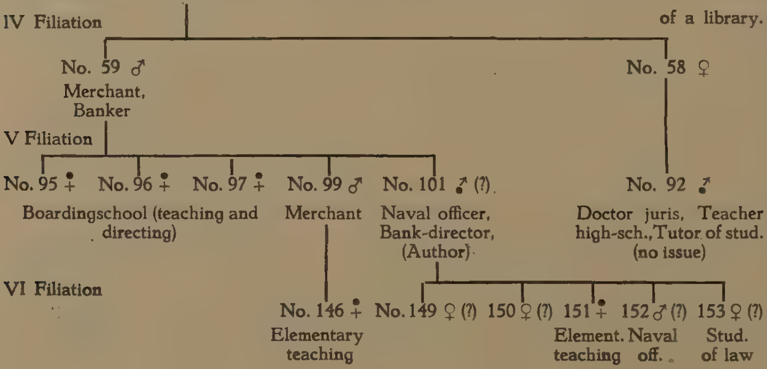
TABLE B.

Parentation No. 1 ♂ (?) Decorator of Delft-pottery
I Filiation No. 2 ♂ (?) „ „ „
II Filiation No. 14 ♂ Headmaster of a boardingschool, author of many treatises for elementary instruction.
III Filiation No. 22 ♀ (?)
IV Filiation No. 36 ♂ (?) Industrial director, station-master.
V Filiation No. 75 ♂ (?) Army-officer, instructor at military school.
VI Filiation No. 113 ♂ Army-officer, afterwards doctor of medicine, university-professor, author.

So here the 3^{d}, 4th, and perhaps the 5th generation are overlapped.

TABLE C.

Parentation No. 1 ♂ (?) Decorator of Delft-pottery.
I Filiation No. 2 ♂ (?) „ „ „
II Filiation No. 16 ♂ (brother of No. 14) lecturer at technical high-school, author.
III Filiation No. 30 ♂ Doctor of medicine, physician, lecturer medical school, director of a library.



Here only the IV Filiation gives no case of positive evidence, In the different falls however where apparently a parent destitute of the property gave rise to offspring provided with it, I presume that the difference between table A and the tables B and C, however striking, is in fact more apparent than real, as the peculiarity in question may have been present in all the included members of the parentela, but may have remained dormant in some of them, being subdued either by outer circumstances or by lack of stimulating reinforcement on the side of the partner in the marriage.

The inclination for instruction, in whatever form it shows itself, may be considered a special manifestation of the general parental instinct for the education of their offspring, which man has in common with the majority of animals. It is only the somewhat heightened tendency and the wider extension of this instinct, which form the remarkable features in the parentela in question. These features become all the more prominent, when we remark that among this rather numerous congregation of consanguineous persons of both sexes, which counts such a high percentage of members devoted to one and the same kind of mental occupation, (and, it should never be forgotten, without the least indication of a family-tradition or other influence of the surroundings), there occurs in the higher generations only one single physician (N⁰. 30). Now even this man, who had a large and successful townpractice, found leasure to teach anatomy at the clinical school then existing in his native town, and to administer a scientific library of a public institute.

It was only after I had remarked with surprise the striking scarcity of medical men among the numerous persons with a university training in the parentela, that I remembered how I myself after having finished my university studies in natural history, had attempted to take up the study of medicine, but had soon abandoned that line of study from want of interest.

Something similar had happened to my father, who entered the university as a medical student, but soon turned to the study of chemistry, to which he devoted his whole further life.

Seen in this light, the fact becomes of interest, that also N⁰. 59, the son of the above-mentioned practitioner, had the intention to follow his father's career, but soon turned to other occupations, for mere dislike of the duties of a medical man. And the same has happened again with N⁰. 161, the only one of the sons of N⁰. 59, who, though he never took up teaching as a profession, yet showed a marked ability for exercising a didactic influence on society in general.

From this course of events in three succeeding generations, it may be fairly concluded, that the choice of social occupation is far more the consequence of inborn qualities than of family-tradition or mere chance.

Tabulating the conclusions from the Parentela as a whole, we arrive at the following numerical results :

Total number of descendants		232
Unfit for consideration on account of early death, youthfulness, mental aberrations or by lack of informations about their mental qualities		95
Remaining for investigation		137
Males	66	
Females	71	
Positive mark attributed to		75
Males	51	
Females	24	

Distribution of these 75 persons over the different categories :

	Instruction	Direction	Study	Authorship	Influencing publ. opin.	Totals
	32	30	60	17	13	152
males	19	27	39	14	12	111
females	13	3	21	3	1	41

So on an average the same person entered into two of these five categories.

By an unsought-for coincidence I obtained the occasion to compare the phenomena of mental heredity in the Parentela van Bemmelen—Blans with the corresponding data in another sequence of descendants. In order to investigate the course of facial resemblance, I studied the entire offspring of a pair of consorts living about the same time (sc. the middle of the eighteenth century) as did Albertus van Bemmelen and his wife Johanna Blans. This second pair were Florentius Camper Florentiusf. and Sara Geertruida Ketting, the parents of the famous Anatomist and Anthropologist Petrus Camper. I intend to publish the results of this investigation in another paper, but here I only wish to give the number of persons who devoted themselves to scientific study and to teaching.

Total number of descendants		174
As yet too young to be taken into account		42
Persons studied		132
Died early		20
Remain		112
Males	63	
Females	49	
Of these 112 the positive mark could be assigned to		40
Males	35	
Females	5	

But among these forty only ten were marked for didactic talents, while the number for study in general was no less than 33.

The second mental gift that I believe can be traced in part of the succeeding generations of the parentela, is one of the artistic endowment, and this has a much more restricted range. It is a very amateurish and dilettante ability for drawing in black and white, without any tendency to using colours, which shows itself in quite the same manner and in an equally restricted grade in two brothers N⁰. 79 and 84, whose three sisters (80, 81, 82) did not possess it in the least, while for the third brother (N⁰. 83) who died long ago in adolescence, the question can no longer be solved.

The special character of this faculty, which could hardly be called a talent, lies in the choice of the subject, which is exclusively the human face, drawn with the purpose of obtaining as faithful a likeness as possible. A second characteristic is the close resemblance between the drawings as far as the manner of execution is concerned, reminding one of the marked correspondence in hand-writing that so often exists between blood-relations.

With regard to heredity the most remarkable feature of this tendency to draw faces is, that it returns in some of the children of these two brothers, namely in the only son of N⁰. 79 (N⁰. 122) and in one of the sons (N⁰. 129) and one of the daughters (N⁰. 130) of N⁰. 84. In the last two however the faculty appears decidedly as a talent, which may possibly be explained by a hereditary influence from the side of the mother (N⁰. 84*a*).

Now neither the father (N⁰. 47) nor the mother (N⁰. 47*a*) of these two brothers showed any natural talent for drawing, or any similar artistic faculty, and no more was this the case with the four grandparents. It is only in the third parentation of the aforesaid brothers that a well-marked case of a similar but far greater aptitude for the drawing of the human face, combined with a very poor sense of colour, shows itself in the brother of the second wife (N⁰. 14*b*) of N⁰. 14. This man was called DANIEL PIERRE GIOTTINO HUMBERT DE SUPERVILLE, who in his time was fairly well known as an artist, especially as a draughtsman. The same faculty appeared to a lesser degree in his brother JEAN, a military engineer, who, while building the fortifications of TUNIS, surveyed and mapped out the ruins of Carthago, in his leasure hours. Of the sister herself no proof exists that she had any talent for drawing.

The father of these two talented brothers was JEAN HUMBERT DE SUPERVILLE, a portrait painter of some renown, few of whose portrait-paintings have come down to us. They show that he must have been a clever draughtsman with but little sense for colour.

With the HUMBERT DE SUPERVILLE's we have passed the limits of the parentela, and consequently lost the contact with its other divisions, containing the descendants of N⁰. 16, the brother of the husband of ANNA FRANÇOISE HUMBERT DE S., and of N⁰. 22 and N⁰. 25, the daughters of N⁰. 14, from his first marriage with N⁰. 14*a*. As far as I know, no marked case of talent for drawing human faces appears in these parts. This contrast between the two branches of the parentela might possibly furnish another argument for attributing the origin of the ability for drawing to the influence of the HUMBERT family. Unfortunately both DANIEL and his brother lost their only child before these had an opportunity of showing that they possessed an artistic gift of any kind.

In the older generations the names HUMBERT and DE SUPERVILLE pass over to different families, but neither in the one nor the other do we meet draughtsmen.

We need not however consider this as a proof, that the talent started as a new variation in the person of JEAN HUMBERT the painter. For when we take into consideration that from this man, that ranks alongside with the members of the first generation of the Parentela, down to his two descendants N⁰. 79 and 84 in the fifth, there is an interval of three generations, in which we do not meet with the slightest trace of a talent for drawing, except in the two side-lines of DANIEL and his brother, the children of both of whom died at an early age, we need not be astonished if we cannot retrace it among still older generations of which we possess very scanty data.

Had our forefathers left us documentary evidence of the mental and bodily peculiarities of their offspring, instead of leaving us a set of names dates and titles, we would see more clearly the reason for the repeated occurrence of hereditary properties and features in succeeding filiations, and in that way get a better insight into the predisposition for certain faculties in our own descendants.

As to general conclusions which might be drawn from special observations like those mentioned in this paper, I think it would as yet be premature to try to frame them. Bio-genealogy is a very young branch of science, and should for the present restrict itself to the collection and critical examination of the immense material afforded by the life-history of the human race. This task in itself is overwhelming, and can only be achieved by careful organising the system of investigation. The various aspects of the subject: the physical and the psychic properties of mankind, must be taken up by different observers, who co-operate by way of distribution of labour.

Before finishing, I should like to make a few remarks on the character of the genealogical unit which I call a parentela, and its value for the science of genetics. It need not be pointed out, that in the majority of cases the differentiation amongst its members will increase with every succeeding filiation. This is caused for the greater part by the continual influx of foreign blood as a consequence of marriages with partners not belonging to the parentela. This progressive differentiation must needs cause the false impression that the influence of the "parents" of the parentela gradually decreases, or, stating it in other words, that the quality of the hereditary properties, introduced by the parental pair, changes in course of time, either for the good or for the bad. That this impression must be false, is conclusively shown by the laws of MENDEL, which have taught us the constant and unalterable nature of the factors, which are the bearers of the hereditary properties. But the impression I mentioned is so strong that we are constantly led astray by it. The force of this misleading conception of heredity becomes clearest to us when we mark the rôle it played in the views of former students of genetics, as yet unacquainted with the doctrine of MENDEL.

GALTON's system e.g., which he applies everywhere in his Hereditary Genius, is entirely based on the gradual incretion and diminution of mental endowment, the first culminating in real genius, the latter leading in a downward direction to the level of mediocrity. On page 83 he gives the well-known table for the "Percentage of Eminent Men in each Degree of Kinship to the most gifted Member of distinguished Families", which should show us that for a hundred of these real geniuses the number of eminent men among their fathers was 26, among their brothers 23, their sons 36, while for the grandfathers, uncles and grandsons these numbers diminished to 7.5, 4.5, and 9.5. GALTON remarks with regard to this result: "The table shows in the most unmistakable manner the enormous odds that a near kinsman has over one that is remote, in the chance of inheriting ability. Speaking roughly, the percentages are quartered in each successive remove, whether by descent or collaterally. Thus in the first degree of kinship the percentage is about 28, in the second about 7, in the third 1½".

GALTON's conception of the influence of foreign blood, entering the parentela, may be clearly understood from what he adds to the above-cited conclusion:

"The table also testifies to another fact, in which people do not commonly believe. It shows that when we regard the averages of many instances, the frequent sports of nature in producing prodigies must be regarded as apparent and not real. Ability, in the long run, does not suddenly start into existence and disappear with equal abruptness, but rather, it rises in a gradual and regular curve out of the ordinary level of family life. The statistics show that there is a regular average increase of ability in the generations that precede its culmination, and as regular a decrease in those that succeed it. *In the first case the marriages have been consentient to its production in the latter they have been incapable of preserving it* 1)".

"After three successive dilutions of the blood, the descendants of the Judges appear incapable of rising to eminence. These results are not surprising even when compared with the far greater length of kinship through which features or diseases may be transmitted".

"Ability must be based on a triple footing, every leg of which has to be firmly planted. In order that a man should inherit ability in the concrete, he must inherit three qualities *that are separate and independent of one another* 1): he must inherit capacity, zeal, and vigour; for unless these

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1) The italics are mine.

PARENTELA ALBERTUS II VAN BEMMELEN (1) 17 × 21 JOHANNA BLANS (1a).

Reference number of person	Reference number of filiation	Reference number of Parents	Family name	Christian names	Born in	Died in	Married	Reference number of consort	Reference number of children	Social position	Natural ability and inclination to				
											Instruction	Giving guidance to others	Study	Authorship	Leadership of public opinion
2	I	1 & 1a	VAN BEMMELEN	Huybert II	1722	1794?	DE VOS, Barbara	2a	11—12						
3	I	1 & 1a	VAN BEMMELEN	Francijntje	1724	1790?									
4	I	1 & 1a	VAN BEMMELEN	Pieternella	1729	1782?									
5	I	1 & 1a	VAN BEMMELEN	Albertus III	1733	1771	1^{0}. RIJKWAART, Jacoba 2^{0}. DE KOK, Marijtje	5a 5b	13—17 18	decorator of Delft pottery					
6	I	1 & 1a	VAN BEMMELEN	Pieter II	1735	1777									
7	I	1 & 1a	VAN BEMMELEN	Johanna	1738										
8	I	1 & 1a	VAN BEMMELEN	Abraham II	1740	1811	VAN DE KEMEL, Marie	8a	19—20	teacher of religion	+				
9	I	1 & 1a	VAN BEMMELEN	Gijsbrecht	1742										
10	I	1 & 1a	VAN BEMMELEN	Elisabeth	1745										
11	II	2 & 2a	VAN BEMMELEN	Albertus IV	1751	1813?									
12	II	2 & 2a	VAN BEMMELEN	Jonathan	1756										
13	II	5 & 5a	VAN BEMMELEN	Albertus V	1753										
14	II	5 & 5a	VAN BEMMELEN	Johannes	1757	1808	1^{0}. SPRUYT, Jacoba 2^{0}. HUMBERT DE SUPERVILLE, Anna Françoise	14a 14b	21—25 26—29	Boardingschool proprietor	+	+	+	+	
15	II	5 & 5a	VAN BEMMELEN	Barbara	1761										
16	II	5 & 5a	VAN BEMMELEN	Abraham III	1763	1822	SLINGERLAND, Dorothea Hendrika	16a	30—31	Dr of science lecturer at technical high-school	+	+	+	+	+
17	II	5 & 5a	VAN BEMMELEN	Andries	1766										

20	II	8 & 8a	VAN BEMMELEN	Abraham IV	1779	1842	HUMBERT, Aletta Jacoba	20a	32—34	clergyman	+		+	+
21	III	14 & 14a	VAN BEMMELEN	Jacoba	1784	1790								
22	III	14 & 14a	VAN BEMMELEN	Antoinette Jacoba	1786	1820	VAN LEEUWEN, Frans	22a	35—39					
23	III	14 & 14a	VAN BEMMELEN		1788	1790								
24	III	14 & 14a	VAN BEMMELEN		1791									
25	III	14 & 14a	VAN BEMMELEN	Johanna Jacoba	1794	1832/40	VAN DER VALK, Harmannes	25a	40—44					
26	III	14 & 14b	VAN BEMMELEN	Jan Frans	1803	1830	DE KEMPENAER, Antoinette Adriana	26a	45—48	headmaster of second. grammar-school	+	+	+	+
27	III	14 & 14b	VAN BEMMELEN	Anna Françoise	1804	1838	SERRURIER Daniel	27a	49—54					
28	III	14 & 14b	VAN BEMMELEN	Frans Anton	1805/6	1840	RIVEAUX, Cornelia	28a						
29	III	14 & 14b	VAN BEMMELEN	Annette Françoise	1807	1869/70								
30	III	16 & 16a	VAN BEMMELEN	Jacobus Albertus	1792	1853	BEGRAM, Henriette Cordula Catharina	30a	55—65	med. dr. pract. lecturer clinical school, librarian	+	+	+	+
31	III	16 & 16a	VAN BEMMELEN	Johannes Cornelis	1796	1824								
32	III	20 & 20a	VAN BEMMELEN	Jacoba Maria	1807	1850								
33	III	20 & 20a	VAN BEMMELEN	Emilie Christine	1808	1853								
34	III	20 & 20a	VAN BEMMELEN	Abraham Leonard	1810	1845				student, theol. afterwards revenue-officer			+	
35	IV	22a & 22	VAN LEEUWEN	Paulus Theod. Marius	1808	1876	1⁰. BREDERO, Cornelia Adriana 2⁰. FISCHER, Wilhelmina	35a 35b	66—69 70—71	clergyman	+		+	
36	IV	22a & 22	VAN LEEUWEN	Johannes Jacobus	1814	1898	MOS, Hendrica Lucretia	36a	72—76	factory-owner aft. station-master		+		
37	IV	22a & 22	VAN LEEUWEN	Daniël Marius	1815	1895	BATENBURG, Hermanna Wilhelmina Dorothea	37a		captain merchant-ship, aft. manager of brick-works		+		
38	IV	22a & 22	VAN LEEUWEN	Ewoud Marius	1817	1878	VAN PELT, Anna Cornelia	38a	77	lawyer, banker		+	+	

PARENTELA ALBERTUS II VAN BEMMELEN (1) 17 × 21 JOHANNA BLANS (1*a*) (*Continued*).

Reference number of person	Reference number of filiation	Reference number of Parents	Family name	Christian names	Born in	Died in	Married	Reference number of consort	Reference number of children	Social position	Natural ability and inclination to				
											Instruction	Giving guidance to others	Study	Authorship	Leadership of public opinion
39	IV	22a & 22	van Leeuwen	François Antoine	1819	18....				banker		+			
40	IV	25a & 25	van der Valk	Maria	1818	1879									
41	IV	25a & 25	van der Valk	Johanna Jacoba	1820	1882									
42	IV	25a & 25	van der Valk	Cornelis	1822	1886									
43	IV	25a & 25	van der Valk	Johannes	1825/6	1887	van Rijswijck, A. F.	43a	78	keeper of lottery-office					
44	IV	25a & 25	van der Valk	Antoinette Cornelia	1832	1832									
45	IV	26 & 26a	van Bemmelen	Pieter	1828	1892	Levoir, Hortense	45a		dr. of law, judge, member of supreme court of judicature	+		+	+	+
46	IV	26 & 26a	van Bemmelen	Anna Françoise	1829	1851									
47	IV	26 & 26a	van Bemmelen	Jacob Maarten	1830	1911	Boeke, Maria	47a	79—84	dr. of chem., university-professor	+	+	+	+	+
48	IV	26 & 26a	van Bemmelen	Adriaan Anthony	1830	1897	Serrurier, Georgine	51	85—87	zoölog., ass. keeper Mus. N. H. Direct. Zoöl. Gardens	+	+	+	+	
49	IV	27a & 27	Serrurier	Daniel	1829	1830									
50	IV	27a & 27	Serrurier	Elisa	1830	18....				governess	+				
51	IV	27a & 27	Serrurier	Georgine	1832	1875	van Bemmelen, Adriaan Anthony	48	85—87						
52	IV	27a & 27	Serrurier	Françoise	1834	1893	Schepper	52a	88—89						
53	IV	27a & 27	Serrurier	Clémentine	1836	1836									
54	IV	27a & 27	Serrurier	Raimond Alphonse	1838	1838									

				Wilhelmina			Theod. Karl Matthieu								
59	IV	30 & 30a	VAN BEMMELEN	Jan Cornelis	1824	1915	ROLAND HOLST, Catharina Agatha	59a	94—103	merchant, dir. Insurance Cy		+			
60	IV	30 & 30a	VAN BEMMELEN	Henriette Jacqueline	1826	1873									
61	IV	30 & 30a	VAN BEMMELEN	Abraham Theodoor	1827	1830									
62	IV	30 & 30a	VAN BEMMELEN	Cordula Judith	1829	1830									
63	IV	30 & 30a	VAN BEMMELEN	Jeanne Madeleine	1830	1832									
64	IV	30 & 30a	VAN BEMMELEN	Willem Lodewijk	1833	1856									
65	IV	30 & 30a	VAN BEMMELEN	Jacobus Albertus	1834	1875				horticulturist			+		
66	V	35 & 35a	VAN LEEUWEN	Frans	1834	1904	BLOM, Johanna	66a	104—107	clergyman	+		+		+
67	V	35 & 35a	VAN LEEUWEN	Cornelis	....	1874				milit. officer		+			
68	V	35 & 35a	VAN LEEUWEN	Cornelia Adriana	1843	1878	DE KANTÈR, Hendrik Machlinus Anthony	68a	108						
69	V	35 & 35a	VAN LEEUWEN	Anna	....	....									
70	V	35 & 35b	VAN LEEUWEN	Paulus Theodorus Marius	....	1883				planter (?)					
71	V	35 & 35a	VAN LEEUWEN	Johan	....	1922	BUNING, Clara	71a	109—110	milit. officer		+			
72	V	36 & 36a	VAN LEEUWEN	Antoinette Catharina	1846	1895									
73	V	36 & 36a	VAN LEEUWEN	Frans Willem	1847	1871				milit. officer		+			
74	V	36 & 36a	VAN LEEUWEN	Arnoldus	1849		STERN, Anna	74a	111	milit. officer		+			
75	V	36 & 36a	VAN LEEUWEN	Daniel Marius	1850	1906	VAN HOYTEMA, Alida Maria Catharina	75a	112—116	milit. officer	+	+			+
76	V	36 & 36a	VAN LEEUWEN	Johannes Jacobus	1854	1904	VAN DRIEL, Petronella	76a	117—118	milit. officer		+			
77	V	38 & 38a	VAN LEEUWEN	Frans	1844		1°. PELTZER, Mary L. C. C. E. C.	77a	119—120	mil. medical officer		+	+		
78	V	43 & 43a	VAN DER VALK	Johanna Jacoba	1822		1°. BUSCHMAN...		121						
79	V	47 & 47a	VAN BEMMELEN	Johan Frans	1859		PAULUS, Adriana Jacoba	79a	122—124	dr. zoöl., univ. professor	+	+	+	+	+

PARENTELA ALBERTUS II VAN BEMMELEN (1) 17 × 21 JOHANNA BLANS (1a) *(Continued).*

Reference number of person	Reference number of filiation	Reference number of Parents	Family name	Christian names	Born in	Died in	Married	Reference number of consort	Reference number of children	Social position	Natural ability and inclination to: Instruction	Giving guidance to others	Study	Authorship	Leadership of public opinion
80	V	47 & 47a	VAN BEMMELEN	Jenny Jacoba	1861	1913				graduate in mathematics, teacher	+		+		
81	V	47 & 47a	VAN BEMMELEN	Anna Françoise	1861	1926								+	+
82	V	47 & 47a	VAN BEMMELEN	Jacoba Maria	1874	1904	1⁰. RAPPARD, Joan Machiel 2⁰. VON LIEBEHERR, Ulrich Bogislav	82a 82b	125—126 127						
83	V	47 & 47a	VAN BEMMELEN	Karel Anton	1867	1892				agriculturist			+		
84	V	47 & 47a	VAN BEMMELEN	Willem	1868		DE JONGH, Soetje	84a	128—132	Dr. of physics, Dir. met. obs., University-lecturer	+	+	+	+	+
85	V	48 & 51	VAN BEMMELEN	Antoinette Jeanne	1859		OP TEN NOORT, Florent Sophius	85a	133—134						
86	V	48 & 51	VAN BEMMELEN	Elisabeth Daniele	1861		PIEPERS, Willem Abrah. Marin.	86a						+	
87	V	48 & 51	VAN BEMMELEN	Catharina Wilhelmina	1865		REUVENS, Louis Caspar	87a	135						
88	V	52a & 52	SCHEPPER	Daniel	1871		NUNES, Elisabeth Josephine Emilie	88a	136—137						
89	V	52a & 52	SCHEPPER	Louise	1874		LOHMANN, ...	89a							
90	V	58a & 58	VON BAUMHAUER	Apollonia	1852		CAMPERT, Jan	90a	138—141						
91	V	58a & 58	VON BAUMHAUER	Henriette Jacoba	1855	1924	1⁰. PENNINK, Math. Mar. Will. 2⁰. FABIUS, Elias	91a 91b							
92	V	58a & 58	VON BAUMHAUER	Paul Matthieu	1859	1920				Dr. of law, tutor of students	+		+	+	
93	V	58a & 58	VON BAUMHAUER	Odilia Emilia	1861	1899				hospital nurse			+		
94	V	59 & 59a	VAN BEMMELEN	Henriette Cordula Catharina	1850		VAN WIJNGAARDEN	94a	142—144						

98	V	59 & 59a	van Bemmelen	Johanna Cornelia	1858	1858									
99	V	59 & 59a	van Bemmelen	Jan Cornelis	1859		Wentholt, Alida Maria	99a	145—148	merchant, Dir. Insurance Cy		+			
100	V	59 & 59a	van Bemmelen	Adriana Saapke	1861	1862									
101	V	59 & 59a	van Bemmelen	Henri Marie	1863		Guépin, Maria Anna	101a	149—153	naval officer, Dir. Insurance Cy		+	+		+
102	V	59 & 59a	van Bemmelen	Cordula Jeanne Madeleine	1865		Smit, F....	102a	154—155						
103	V	59 & 59a	van Bemmelen	Theodora Car. Wilh. Matthea	1866					hospital nurse		+			
104	VI	66 & 66a	van Leeuwen	Johan Frans Jozef	1860	1920	Goldsbury, Edith	104a		merchant (U.S.A.)					
105	VI	66 & 66a	van Leeuwen	Cornelia Adriana	1863					hospital nurse				+	
106	VI	66 & 66a	van Leeuwen	Paulina Theodora Maria	1866		Boerma, Roelf Herman	106a							
107	VI	66 & 66a	van Leeuwen	Ernst George	1869		Hugenholtz, Elisabeth	107a	156—159	agriculturist (U.S.A.)					
108	VI	68a & 68	de Kanter	Cornelis Machlinus	1869		Betz, Maria Henriette	108a	160—162	quartermaster-general		+	+		
109	VI	71 & 71a	van Leeuwen	Paul Tjaarda Hendrik	1899		Solzer, Sophia Johanna Wilhelmina	109a	163—166	colonial civil serv.					
110	VI	71 & 71a	van Leeuwen	Louise			van Lissa.....	110a	167—168						
111	VI	74 & 74a	van Leeuwen (Storm v. L.)	Anna C....P....C	1895		de Sturler, Carel Adriaan	111a	169—171						
112	VI	75 & 75a	van Leeuwen (Storm v. L.)	Henriette Lucretia Wijnanda	1880		Kuyper, Guillaume	112a	172—174						
113	VI	75 & 75a	van Leeuwen (Storm v. L.)	Willem	1882		Klerk de Reus, Anna Judith	113a	175—178	mil. officer, dr. of med., univ. prof.	+	+	+	+	+
114	VI	75 & 75a	van Leeuwen (Storm v. L.)	Hermanna W. O....	18..		Bakker, H....	114a	179—181						
115	VI	75 & 75a	van Leeuwen (Storm v. L.)	Arnold	18..		1^0. v. d. Made, Thile 2^0. Cramer, W....	115a 115b	182 183	antique dealer					
116	VI	75 & 75a	van Leeuwen (Storm v. L.)	Alida Marie Catharina	18..		Cramer, W.....	116a							
117	VI	76 & 76a	van Leeuwen (Storm v. L.)	Hendrika Lucretia	1887		Terwogt, Joh. H. L,	117a	184	teacher of music	+		+		
118	VI	76 & 76a	van Leeuwen (Storm v. L.)	Petrus Laurentius	1888		van Lokhorst, Johanna	118a	185—190	planter		+			

PARENTELA ALBERTUS II VAN BEMMELEN (1) 17 × 21 JOHANNA BLANS (1a) (*Continued*).

Reference number of person	Reference number of filiation	Reference number of Parents	Family name	Christian names	Born in	Died in	Married	Reference number of consort	Reference number of children	Social position	Natural ability and inclination to				
											Instruction	Giving guidance to others	Study	Authorship	Leadership of public opinion
119	VI	77 & 77a	van Leeuwen (Storm v. L.)	Anna	....		Meijeringh, Willem	119a	191						
120	VI	77 & 77a	van Leeuwen (Storm v. L.)	Carolina	....		Valois, H.....	120a	192						
121	VI	78a & 78	Buschman	Petronella Berendina	1904										
122	VI	79 & 79a	van Bemmelen	Jacob Maarten	1898		Kluit, Clara Adr. Petr.	122a	193—194	Dr. of law, pr. tutor teach. second. sch.	+	+	+	+	+
123	VI	79 & 79a	van Bemmelen	Maria	1901		Mahr, Adolf Marie	123a	195—196	teacher of music	+		+		
124	VI	79 & 79a	van Bemmelen	Joziene Berendina	1904		Antonius, Julius	124a	197—198	dipl. graduate for Univ. education			+		
125	VI	82a & 82	Rappard	Maria Mathilde	1889		1^0. v. Drumpt, Jan 2^0. Wilson, John	125a 125b	199—200						
126	VI	82a & 82	Rappard	Elizabeth Louise	1890		1^0. Polak Daniels Eduard 2^0. v.d.Stadt, Jacob Adriaan	126a 126b	201 202						
127	VI	82b & 82	von Liebeherr	Bogislav Ulrich	1903	1925				agriculturist			+		
128	VI	84 & 84a	van Bemmelen	Daniel Johan	1899		Smit, Emilia	128a	203	teacher	+		+	+	+
129	VI	84 & 84a	van Bemmelen	Hermina Wouthera	1902		IJzerman, Egbert	129a		teacher	+		+		
130	VI	84 & 84a	van Bemmelen	Reynout Willem	1904					dr. of geology, assistant-lecturer Techn. highsch.	+		+	+	
131	VI	84 & 84a	van Bemmelen	Frans Johan	1913	1915									
132	VI	84 & 84a	van Bemmelen	Anna Françoise	1916										
133	VI	85a & 85	Op ten Noort	Adriaan Anthony	1886										

138	VI	90a & 90	Campert	Petrus Jan Remko	1874	1921	van Hall, Johanna	138a	210—213	medic.practitioner			+	
139	VI	90a & 90	Campert	Dorothea Wilhelmina	1875		de Ranitz, H.	139a	214—215	teacher	+		+	
140	VI	90a & 90	Campert	Johanna Berendina	1879		Waszink, Jan Hendrik	140a	216—218	assistantBotanical Laboratory	+		+	
141	VI	90a & 90	Campert	Henriette Jacoba	1882					teacher	+		+	
142	VI	94a & 94	v. Wijngaarden	Adrienne Marthe Henr.	1877	1908								
143	VI	94a & 94	v. Wijngaarden	Johanna Cath. Corn.	1880		Wagtho, Johan Willem	143a	219—220					
144	VI	94a & 94	v. Wijngaarden	Catharina Agatha	1883		de Jongh, Will. Hendr. Dan.	144a	221—222					
145	VI	99 & 99a	van Bemmelen	Jan Cornelis	1892									
146	VI	99 & 99a	van Bemmelen	Meike Alida	1893					teacher	+		+	
147	VI	99 & 99a	van Bemmelen	Ludolf Reinier	1894									
148	VI	99 & 99a	van Bemmelen	Henri Marie	1896									
149	VI	101 & 101a	van Bemmelen	Maria Anna	1894					hospital nurse			+	
150	VI	101 & 101a	van Bemmelen	Cordula Cath. Agatha	1896		Quarles v.Ufford, Louis Jacques	150a	223—224					
151	VI	101 & 101a	van Bemmelen	Catharina Marianne	1900					teacher	+		+	
152	VI	101 & 101a	van Bemmelen	Henri Marie	1902		Nuboer. Ida Elisabeth	152a		naval officer		+	+	
153	VI	101 & 101a	van Bemmelen	Theodora Cath. Wilh. Matthea	....					bachelor of law			+	
154	VI	102a & 102	Smit	Cordula Frédérique	1890		Remmelts, Remmelt	154a	225—227					
155	VI	102a & 102	Smit	Wilhelmina Marie	1901									
156	VI	107 & 107a	van Leeuwen	Frans	1897									
157	VI	107 & 107a	van Leeuwen	Anna	1901		Ward, Paul							
158	VI	107 & 107a	van Leeuwen	Betsy	1913									
159	VI	107 & 107a	van Leeuwen	Ernest	1913									

PARENTELA ALBERTUS II VAN BEMMELEN (1) 17 × 21 JOHANNA BLANS (1*a*) (*Continued*).

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											Instruction	Giving guidance to others	Study	Authorship	Leadership of public opinion
160	VII	108 & 108a	DE KANTER	Susanna Antoinetta	1894		VAN HOLTHE, Leo Johan Dignus	160a	228						
161	VII	108 & 108a	DE KANTER	Cornelia Adriana	1897		VAN STRAATEN, Ger. Lamb.	161a	229—231						
162	VII	108 & 108a	DE KANTER	Godefrieda Hendrika	1901		V. D. POLL, Hendrik Willem	162a	232						
163	VII	109 & 109a	VAN LEEUWEN	Tjaarda Paul	1915										
164	VII	109 & 109a	VAN LEEUWEN	Johan Willem	1917										
165	VII	109 & 109a	VAN LEEUWEN	Gerrit Albert	1920										
166	VII	109 & 109a	VAN LEEUWEN	Nancy Johanna	1921										
167	VII	110a & 110	VAN LISSA	Paula											
168	VII	110a & 110	VAN LISSA	Johan											
169	VII	111a & 111	DE STURLER	Anna Suzanna	1916										
170	VII	111a & 111	DE STURLER	Carolina Wilhelmina	1917										
171	VII	111a & 111	DE STURLER	Sara Adriana Dorothea	1921										
172	VII	112a & 112	KUYPER	Alida Maria Catharina	1907					stud. of medical chemistry			+		
173	VII	112a & 112	KUYPER	Johanna Hendrika	1909										
174	VII	112a & 112	KUYPER	Maria Eva	1914										
175	VII	113 & 113a	VAN LEEUWEN	Anne Judith	1906					graduate for med. chemistry			+		

				Christina											
180	VII	114a & 114	BAKKER	Theodora Agatha Mar. Ant.	1915										
181	VII	114a & 114	BAKKER	Bernard Anton	1918										
182	VII	115 & 115a	VAN LEEUWEN	Alida Maria Cath.	1922										
183	VII	115 & 115b	VAN LEEUWEN	Arnold	....										
184	VII	117a & 117	TERWOGT	Anna Bertha	1926										
185	VII	118 & 118a	VAN LEEUWEN	Johannes Adriaan	1912										
186	VII	118 & 118a	VAN LEEUWEN	Elisabeth Bertha	1916										
187	VII	118 & 118a	VAN LEEUWEN	Hendrik Arnoldus	1918										
188	VII	118 & 118a	VAN LEEUWEN	Petrus Jacobus	1920										
189	VII	118 & 118a	VAN LEEUWEN	....											
190	VII	118 & 118a	VAN LEEUWEN	Willem	1926										
191	VII	119a & 119	MEYERINGH	Sjoerd	1914										
192	VII	120a & 120	VALOIS	Mary	....										
193	VII	122 & 122a	VAN BEMMELEN	Johan Frans	1923										
194	VII	122 & 122a	VAN BEMMELEN	Adriaan Gerard Oswald	1925										
195	VII	123a & 123	MAHR	Gustav Adolf Marie	1922										
196	VII	123a & 123	MAHR	Hildegarda Adriana Jacoba	1926										
197	VII	124a & 124	ANTONIUS	Dorothea Beata	1925										
198	VII	124a & 124	ANTONIUS	Liselotte Helene	1927										
199	VII	125a & 125	VAN DRUMPT	Daniel Johan Ulrich	1911										
200	VII	125a & 125	VAN DRUMPT	Jacoba Simonette	1912										
201	VII	126a & 126	POLAK DANIËLS	Marianne Elisab.	1915										

PARENTELA ALBERTUS II VAN BEMMELEN (1) 17 × 21 JOHANNA BLANS (1a) (*Continued*).

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											Instruction	Giving guidance to others	Study	Authorship	Leadership of public opinion
202	VII	126b & 126	van de Stadt	Maria Helena	1925										
203	VII	128 & 128a	van Bemmelen	Beatrys Helena	1926										
204	VII	134a & 134	Sickenga	Jacob	1918										
205	VII	134a & 134	Sickenga	Antoinette J.	1922										
206	VII	134a & 134	Sickenga	Wilhelmina F. C. C. I.	1924										
207	VII	134a & 134	Sickenga	Florent Sophius	1927										
208	VII	135a & 135	Joosting	Else Petra	1920										
209	VII	135a & 135	Joosting	Peter Erik	1927										
210	VII	138 & 138a	Campert	Jan Remco Theodoor	1902					student of arts			+		
211	VII	138 & 138a	Campert	Herman Marie	1904					student of commercial economy			+		
212	VII	138 & 138a	Campert	Apollonia Wilhelmina	1905					graduate for elem. teach., hosp. nurse			+		
213	VII	138 & 138a	Campert	Anne Marie	1907					graduate sec. sch., shorthandwriting			+		
214	VII	139a & 139	de Ranitz	Jan	1909					student of architecture			+		
215	VII	139a & 139	de Ranitz	Herman	1910					stud of medicine			+		
216	VII	140a & 140	Waszink	Jan Hendrik	1908					student of classics			+		
217	VII	140a & 140	Waszink	Paul Mathieu	1911					pupil of secondary school			+		
218	VII	140a & 140	Waszink	Herman	1914										

221	VII	142a & 142	DE JONGH	Willem	1905									
222	VII	142a & 142	DE JONGH	Eric Eduard	1907									
223	VII	149a & 149	QUARLES V. UFFORD	Louis Jacques	1919									
224	VII	149a & 149	QUARLES V. UFFORD	Marie Anne	1921									
225	VII	154a & 154	REMMELTS	Nicolaas	1922									
226	VII	154a & 154	REMMELTS	Justine	1923									
227	VII	154a & 154	REMMELTS	Maria	1926									
228	VIII	160a & 160	VAN HOLTHE	Jan Cornelis	1923									
229	VIII	161a & 161	VAN STRAATEN	Herman Cornelis	1922									
230	VIII	161a & 161	VAN STRAATEN	Gerard	1924									
231	VIII	161a & 161	VAN STRAATEN	Robert	1925									
232	VIII	162a & 162	VAN DE POLL	Hendrik Albrecht	1925	1925								

three, or, at the very least, two of them, are combined, he cannot hope to make a figure in the world. The probability against inheriting a combination of three qualities not correlated together, is necessarily in a triplicate proportion greater than it is against inheriting any one of them".

This remarkable quotation proves, that GALTON, though he was naturally unacquainted with MENDEL's discoveries, had a clear insight into the mechanism of heredity, in so far as he ascribed its phenomena to the influence of "qualities that are separate and independent of one another". But as to the real nature of these "qualities", he probably did not consider them as we do now a days, that is to say as primary, unalterable genotypical factors passing from generation to generation without changing their constitution, and only varying in their phenotypical effect under the influence of other factors, internal as well as external.

Had he been able to consider things from this point of view, he would probably not have chosen cases of perfect genius as the central-points, from which he mapped out his lines of ascent and descent, as it is clearly obvious that genius in itself is no entity in the sense of a hereditary factor, but only a popular qualification for the highest grades of a combination of certain valuable mental properties. For us GALTON's choice of his points of issue seems as queer as would be the method of a modern student of genetics, who based an investigation about the rules of heredity for eye-colour on the very exceptional cases of people with one brown and one blue eye, or an enquiry into the duration of human life on those of centinarians.

Although WOODS' book appeared in 1906, when the new light on questions of heredity had already appeared, and notwithstanding his acquaintance with the phenomenon of "Alternative inheritance", he still adheres to GALTON's "law of the Average Contribution of Each Separate Ancestor to the Total Inheritance of the Offspring". In WOODS' eyes, remoteness of an ancestor is forcibly equivalent with reduced influence on the hereditary constitution of the descendant.

Yet, it is easy tọ cite a profusion of instances, where he is forced to acknowledge the reversion on a remote forefather, or the reappearance of a property which seemed longsince extinct. Compare e.g. p. 51 : "The tendency to mental unbalance (insanity) in the house of HANOVER, where it showed itself in only a few examples, may be considered to be probably a reversion to the old Palatine insanity at the time of the STUARTS".

It is, however, in the cases where WOODS points out the regular and uninterrupted repetition of fairly normal mental properties, which families of royal blood have in common with all other people of whatever standing in social life, that he furnishes us with the best examples of hereditary propagation of special traits, without any regard to the distance towards the first member in the pedigree that showed them, and likewise to their comparative grade of accomplishment or to their social merits. The most interesting case in this connection is what he tells us about .the distinct literary tastes of the House of BRUNSWICK. "Among the forty-five" (members of the first five generations) "here studied, we find...... twelve who were either authors, or showed distinct literary tastes"...... "in the direct male line". Only a few lines further he adds: "From now on, literary...... ability was absent in the House of BRUNSWICK, and the story of the last two generations is but the sad extinction of a once illustrious and honored family...... A congenital psycho-neurosis made its appearance, the origin of which may be surmised, though not incontestably proved".

Though I must confess, that I am unacquainted with the literary products of these twelve authors of royal blood, I yet venture to assert, that, had they not belonged to a "once illustrious and honored family", these products would not in themselves have been of sufficient merit to bring the fame of their authors down to our present days. Nor is literary taste and an irrepressible vocation for the composition of works of fancy, in any way connected with the special traditions or the favourite sports of members of royal families. On the contrary, we need not fear contradiction when we assert that in general such an active partaking in artistic or scientific achievements of commonplace people is a great exception among them, which is far from being regarded with a favourable eye by the majority of their standing. So WOODS' argumentation that such a series of generations of literarely-endowed persons of royal extract can only be explained by the pure force of heredity, is perfectly superfluous. The phenomenon only proves once again that these persons, mentally as well as bodily, are in no way differently constituted from the bulk of mankind.

Physiology. — *On the Physiology of Reflex-Standing.* By Dr. G. G. J. RADEMAKER. (Communicated by Prof. R. MAGNUS).

(Communicated at the meeting of June 25, 1927).

When a few years ago an examination was started about the functions of the cerebellum, the relative literature was, of course, consulted. According to LUCIANI decerebellated animals display astasia, although they are able to stand, which is somewhat a contradictio in terminis. Also according to BECHTEREW these animals display disturbances of standing: "astasia consequent on a disturbance of static co-ordination".

Now we had to ask the question: what do the decerebellated animals lack for standing; from this question two others arose, viz. what is required for standing, and how is normal standing brought about.

These questions seem to me to be of great importance also for the clinic. When still attending patients in the clinic I asked myself how is it that when a patient lying on his back is examined for the tonus of the leg-muscles by grasping the leg above the ankle and flexing it passively, this flexure can rather easily be effected, even when the leg-muscles are hypertonic, whereas a standing normal person tolerates a heavy pressure upon the shoulders, without any flexion of the legs. The legs of standing dock-workers can carry from 200 to 300 kg. plus the weight of the trunk. This is more or less a mechanical action, not exclusively a voluntary one, and is performed unreflectingly.

Therefore, the recent manuals of Physiology (a.o. that of STARLING) and of Neurology were consulted on the subject of standing, but next to nothing was found about it, in the Manuals of Neurology only the familiar pictures of disturbances of standing and of the efforts to get up from a lying position in cases of progressive muscular dystrophia.

In other, especially in the older text-books of Physiology, we find laborious treatises on the standing postures, on the position of the joints in a standing attitude, on the course of the lines of gravity, and on the function of the bones, ligaments and tendons. In these treatises the muscles are altogether ignored. This is because the writers thought that muscular exercise always brought fatigue. Standing for hours being possible without great fatigue, an operation of the muscles was considered out of question.

It was especially DUCHENNE DE BOULOGNE who pointed out that the muscles play a significant part in standing, as with paralysis, or disease of the muscles standing is hampered very much, even often rendered impossible. He also maintained that in the normal standing attitudes all the leg-muscles, extensors as well as flexors, and abductors as well as adductors, feel hard and stretched.

In 1896 SHERRINGTON reported that when an animal had been decerebrated, (i.e. when the mesencephalon had been severed transversely between the anterior and the posterior corpora quadrigemina, consequently caudal to the red nuclei), it was quite rigid, and that when the rigid animal was put on its legs, the legs could support the trunk, which the leg of the spinal animal cannot. According to SHERRINGTON the decerebrated animal can stand, it exhibits, in contradistinction to the spinal animal "standing-reflexes", consequently "reflex-standing" and possesses static tonus.

Formerly it had already been demonstrated by FLOURENS, MAGENDIE, and others, notably by GOLTZ, that also thalamus-animals can stand and, it would seem, with a normal distribution of tonus. So the thalamus-animal also displays a reflex-standing, but much more normal. MAGNUS pointed out that the posture of decerebrated animals, with the maximally stretched joints of the legs and the excessive extensortonus, is a grotesque posture.

On investigation the "reflex-standing" of a decerebrated animal exhibits such numerous and pronounced deviations, that the question rises whether it may in strictness be called standing.

I. In the first place the distribution of tonus is quite different. Not only that in decerebrated animals the extensor tonus is abnormal, but also that the flexors lack tonus altogether or nearly, while with normal standing posture, as we have already seen, *all* the leg-muscles are contracted. SHERRINGTON maintains that the standing attitude is chiefly brought about by the extensors, which according to him are the "antigravity", the muscles counteracting the gravity. That this is not always quite true, can be stated for animals as for men. When the forelegs of a dog are slightly retro-flexed, the triceps brachii feels quite flaccid. In posture with trunk bent forward the rectus femoris in man is soft, while the patella can be easily moved to and fro. Furthermore standing, as well with stretched as with semi-flexed legs is possible when the quadriceps are entirely paralyzed on either side, The literature contains records of several cases in which this was observed.

II. Another disturbance displayed by the decerebrated animal is that it does not put its legs in the right position for standing. When the animal is held freely in the air by head and tail, and is then let down till the toes touch the floor, the legs are in a wrong attitude, they rest on the dorsal side of the toes, and the animal does not put them in the right position. When the animal is lowered still more, the legs will slip backward, and finally they will be retroflexed under the body.

III. If we put the legs in the proper position, and then move the animal to and fro antero-posteriorly or sideways, it does not shift the legs ; the legs do not make correcting-movements. Also when putting the legs passively in an abnormal position, there is no trace of correction noticeable, i.e. the animal does not lift up its leg to put it down in the right position. (When flexed passively the toes recoil to the stretched-position on being let loose.)

IV. If we do not help the standing animal sideways, it overturns on the slightest push, or the slightest movement it makes, without any active efforts to prevent it, and it remains lying on its side. The righting-function, the capacity of keeping and resuming the erect position, is totally absent. (Of Magnus's righting reflexes only the neck righting reflexes are still present).

V. The legs of the decerebrated animals are not only strong in a standing position : they not only show extensor-tonus when the animal is "standing" but also when it is put in side- or dorsal position.

With dorsal position the extensor-tonus is often even stronger under the influence of the tonic labyrinthine reflexes. The extensor-tonus, therefore, is not caused by standing : it is not brought about by static relations. According to Liddell and Sherrington in the standing of decerebrated animals myotatic reflexes come into play, i.e. reflexes evoked by stretching of the muscles consequent on the weight of the body. These myotatic reflexes, however, cannot be the cause of the extensor-tonus in the lateral position of the decerebrated animal.

VI. And lastly the decerebrated animal placed on a board inclining towards one of the sides, does not exhibit any changes in the distribution of tonus ; the position of the legs is not adjusted to the static relations.

The so-called static tonus of decerebrated animals, therefore, is neither evoked by the static relations, nor does it accommodate itself to the static relations ; it is even mostly more pronounced in the dorsal position.

The decerebrated animal stands more or less like a toy-animal on four stiff pillars. Of course, this comparison also halts : as the tonus of the leg-muscles varies with the change of the head-position, on labyrinth-stimuli etc.

What are the requirements for normal standing ?

In the first place that the legs should be placed in the proper position. This may be effected by two sorts of stimuli :

First by optic stimuli,

Secondly by stimuli of different areas of the body surface.

The influence of the optic stimuli can be seen, when we hold an animal freely in the air, and then approach the head to the edge of a table. As soon as the head is at a short distance from it, the animal put down its legs. This

as the head is at a short distance from it, the animal puts down its legs. This

these stimuli also play an important rôle in men is evident from the behaviour of tabetici with open and closed eyes. The action of stimuli emanating from the body-surface may be seen when a blindfolded animal is suspended in the air, and the lower jaw is then brought in contact with a tableboard. The forelegs are placed directly beside the muzzle, then they are stretched and the animal is supported by them. Also when the dorsal side of the leg touches a board, the legs will be placed in a normal position on the board. Likewise pads of the hindlegs will be placed on the board when the point of the tail touches them.

These "placing-reflexes" consequent on contact are "conditioned reflexes" in PAWLOW's acceptation. If the dog is held up free in the air, with the head upward, and then the tail is touched by the hand, the hindlegs will also be stretched, but subsequently become flexed again, because the pads do not touch the board. If we repeat this five or six times, thus fooling the animal, no stretching comes forth any more. The reflex consequent on touching of the tail may be evoked again by lowering the animal a couple of times, so as to make the tail and subsequently the hindlegs touch the board. This property of being "conditioned" suggests, that we have to do here with cerebrum-reflexes. Two facts are in perfect harmony with this, viz. in thalamus-animals and also in decerebrated animals these reflexes are absent, and secondly blindfolded dogs with unilateral extirpation of the cerebrum put down only the legs of the side of the extirpation, while the contralateral legs remain hanging in the air. The unilateral absence of these reactions becomes evident also when these animals are placed on a grating and the legs are pulled between the bars. Then the legs on the side of the extirpation are placed directly on the bars, the contralateral ones retain their position between the bars. On the contrary blindfolded decerebellated animals place their legs in the right way on touching a floor, and place them also correctly on the bars of the trellis, anyhow some time after the extirpation. The first week or weeks they also often show disturbances.

As regards man, first of all we know, that in nurslings the reactions are lacking in the first half year. If we bring the posterior part of their body in contact with the floor they do not put their legs down in the right way, they lift them up. Every neurologist knows that these reactions can also be absent in pathological conditions.

In the second place an adequate supporting tonus is required for standing.

If a normal or a decerebellated animal is placed in a dorsal position, or is held up free in the air, the legs present a posture, and a distribution of tonus--relation quite unfit for standing. The legs are then more or less flexed and the resistance against passive flexion is in that case only slight. If, however, the animal is put on its legs, they are stretched at once and have changed into firm pillars that can hardly be bent. The legs then exhibit a very marked "supporting-tonus". All the muscles are protruding, and flexors as well as extensors, abductors as well as adductors feel hard and tight. The term supporting-tonus was chosen to distinguish it from the so-called static tonus of the decerebrated animals.

The following four factors play a rôle in the origin of the supporting tonus.

I. *The position of the distal parts of the extremities:* at the forelegs the position of wrist and fingers, at the hindlegs that of the toes.

We have seen already in what way the hands and the feet were placed in the right position. This change of position elicits numerous reflex-contractions, e.g. with extension of wrist and fingers reflex-contractions of the muscles of the upper arm, shoulder and shoulderblade appear, by which elbow and shoulder are extended and are fixed in this position. This fixation continues so long as the wrist is stretched passively. Likewise stretching of the toes of the hindlegs engenders extension of hip and knee, whereas with flexion of the toes the resistance against passive flexion of these joints disappears, and often reflex bending of knee and trip appears, (positive and negative supporting reactions).

These reflex-contractions can readily be felt. They were, moreover, demonstrated by SCHOEN at muscles detached from their insertions (at a so-called Sherrington-preparation).

These reactions are effected by proprioceptive stimuli. With passive extension of the wrist, fingers and toes the flexors are stretched, by which stimuli are elicited in these muscles, which by way of the central nervous system contract the other muscles of fore-, resp. hindleg.

It is quite in keeping with the proprioceptive character of these reactions, that they are still present in legs anaesthetized by severing the cutaneous nerves, and that on the contrary they disappear after severing the posterior roots of the spinal cord. After transverse section of the spinal cord extension of the wrist still produces some extension of the proximal joints, but this stretched posture is not fixed tonically. Also with decerebrated animals these reactions exhibit anomalies; with passive flexion of the distal joints fixation does not disappear completely, as the legs will still retain a distinct rigidity, a distinct resistance against passive flexion. The positive supporting reaction, of course, cannot be ascertained from the stretched, stiff legs. With thalamus animals the positive, and the negative supporting reactions are normal, with decerebellated animals they are excessively marked. Also in men similar mechanisms are operating. The nurses of the Salpêtrière in CHARCOT's time had already observed that extensor cramps of the legs ceased when the toes were flexed with force. Afterwards this influence of flexion of the toes on the extensor tonus in cases of hemi-, and paraplegia, and with tranverse myelitis was described by PIERRE MARIE and FOIX as "phénomène des raccourcisseurs". It is known that when the foot is moved dorsad, it also yields reflex changes of the tonus distribution. I need only to remind the reader of the fact that it often evokes foot-clonus.

II. Secondly *touching the footsole* plays a rôle in evoking the supporting tonus.

A normal dog or a decerebellated dog in dorsal position keeps its hindlegs flexed. Now if we touch the dorsal side of the foot, the flexed position persists. If, however, we touch the sole, the leg is stretched at once to its full length, and is fixed in this stretched position. Then all the muscles are contracted, protrude and feel hard. The ankle, knee and hip-joint are also entirely fixed. The pelvis follows every passive movement of the upperleg. If we touch the sole of the flexed hindleg and draw the touching finger back, the leg will follow as if attracted by magnetic force. For this reason this touching reaction i.e. a reaction to exteroceptive stimuli was baptized magnet-reaction. The forelegs also show this reaction ; they are stretched when the sole is touched, and elbow, shoulder, nay even the shoulderblade are completely fixed. Not only all the muscles of the extremities exhibit reflex-contraction, also the muscles of the back, and, under certain circumstances also the cervical muscles. This influence on the muscles of the back and the neck is also demonstrable with dorsal position.

The contractions caused by touching the sole are *tonic*, they persist all the time the touching continues.

Of course, there is no magnet-reaction in anaesthetized legs, it is an exteroceptive reaction ; nor is there any after severance of the posterior roots of the spinal cord. After transverse section of the spinal cord it is also absent, anyhow touching the soles does not give tonic contractions, which persist so long as the touching continues. (With spinal animals touching of the sole often results in a short stretching followed by flexion, the so-called extensor-thrust of SHERRINGTON.) It was observed that neither with decerebrated animals, nor with thalamus-animals touching of the soles was followed by the magnet-reaction ; it was remarkable, however, that after unilateral extirpation of the cerebrum, a tonic stretching appeared when the soles were touched, as well at the homolateral as at the contralateral paws.

In decerebellated animals magnet-reaction is ever extremely vivid and yields almost maximal extension and fixation. Also the muscles of the back react excessively in decerebellated animals on the touching of the sole, and the concave back of the dog held up free in the air by the head and the tail, is not as with the normal dog bent straight but arched.

Man has not yet been examined systematically for the existence of magnet-reaction. We do know that in normal adults touching of the soles is not followed by a stretching of the leg, and also that in nurslings of 1—6 months no stretching or supporting tonus appears when the soles are brought in contact with the floor. I regret to say that I have not yet been in a position to ascertain the consequences of contact of the soles in children that have just learned to stand or walk. If we touch the hollow of the hand of a young child, a "clinging reflex" appears just as with monkeys. In pathological conditions this reflex also occurs in adults a.o. often in cases of dementia. These patients appear to me just fit subjects to observe the consequences of the touching of the soles, just as patients with cerebellar atrophy, because in decerebellated animals the magnet-reaction is particularly vivid and unrestrained.

Thirdly *pressure on the soles* also plays a rôle in evoking the supporting tonus.

This pressure will act first of all as exteroceptive stimuli, as the magnet-reaction ; only the intensity of the exteroceptive stimuli is in this case greater. However, pressure also exerts an influence on the anaesthetized paw. If the terminals of the paws are first brought in a standing position. and subsequently pressed on the sole, this pressure on the sole will intensify the stretching of the flexors of fingers, wrist and toes and the same reactions will appear as with passive placing of the terminals in the position as with the standing animal. The reactions, however, will be stronger. The result will be stretching in all the joints and still stronger fixation of the stretched position of the limb.

If the limb is already quite stretched the pressure on the sole will stretch besides the named flexors, also the extensors of elbow and shoulder, resp. of ankle, knee and hip, and through the stretching of these muscles not only these, but also others will show reflex contractions. So SCHOEN demonstrated that on the stretching of the m. triceps bracchii not only the triceps itself reacted with a contraction, but a.o. also the biceps and the supraspinatus.

In the standing animal a counterpressure is exerted on the soles equal to the bodyweight. Now, if the back is loaded, the counterpressure increases, so that all the above-named muscles will be stretched still more and hereby bring about still stronger reflex-contractions. LIDDELL and SHERRINGTON demonstrated that the reflex-contractions, evoked by stretching, will be stronger, first of all according as the stretching is more intense, and secondly as the incipient tension of the muscles is larger. In this way the adaptation to the loaded weight is effected.

The pressure on the soles also augments the tension of the muscles of the vertebral column, and, under certain circumstances, also of the cervical muscles.

In man this influence on the dorsal spinal muscles is also noticeable, notably when palpating the m. sacrolumbales, while standing alternately on the left and right leg.

After transverse section of the spinal cord pressure on the soles gives neither stretching nor a tonic contraction of the muscles. In decerebrated animals pressure on the sole causes tonic contraction only of extensors ; the flexors do not react. In thalamus animals pressure on the soles produced a tightening of flexors and extensors, but the fixation of the hindlegs most often did not become so strong as in normal or decerebellated dogs. In the latter fixation of the joints is very much intensified by pressure.

If a normal or a decerebellated dog is placed with one leg on a balance, it appears that the hindleg can bear a counterpressure of 1½ times the bodyweight, the foreleg even more than double that weight, without flexing.

IV. Finally *the mutual influence of the positions of the proximal joints* come into play.

If one of the proximal points, elbow or shoulder, resp. knee or hip, is stretched passively or actively, other joints will also stretch and are fixed in stretched position. If e.g. of a dog the upperarm is moved forward, the wrist and the elbow will also be stretched and these joints cannot be flexed passively in that case. Conversely, passive flexion of one of the proximal joints is attended with flexion of the other joints and fixation in flexed position. With passively flexed shoulder, elbow and wrist cannot be stretched completely.

This mutual influence is principally, perhaps entirely, attributable to the anatomic relations. Reflex influences of one muscle on the other are not distinctly observable. The appearance of this phenomenon depends, however, on an adequate tonus of the muscles. In spinal dogs it is still present ; stretching of a proximal joint is still attended with stretching of others, mostly, however, with smaller fixation. On the other hand, it is no longer observed in maximal, deep narcosis, or only concomitant stretching occurs without fixation. This mutual influence of the joints is answerable for the fact that a fully stretched leg is more difficult to flex than a semi-flexed one.

In the foregoing we discussed :

1⁰. how the legs are placed in the proper position for standing ;

2⁰. in what way the supporting tonus is evoked ; and

3⁰. how the supporting tonus is adapted to the loading.

It still remains for us to ascertain what factors play a rôle in the adaptation to the static relations, to the changes of the plane of the floor. Only the principal factors will be discussed briefly :

I. *The position of the head.*

If we place a normal (or better still a decerebellated) dog in dorsal position and hold the muzzle vertically upwards or ventral, the hindlegs will, on touching the soles, display a marked magnet reaction, and pressure on the soles is also followed by the occurrence of intense stretching and a strong resistance against passive flexion. If, now, while pressing the soles we move the head in dorsal direction, we feel the resistance slacken and ultimately disappear when the muzzle sinks below the horizontal. Then the hindlegs will twitch and neither touching nor pressure on the sole will evoke a stretching any more. Also in the standing animal this influence of the position of the head and neck on the supporting tonus of the hindlegs, is evident. With a posteriorly upward position of the head the supporting tonus of the hindlegs is weak, and it is strong with ventral flexion of the head and the neck. Also the supporting tonus of the forelegs changes in this process, but in an opposite sense. When raising the head it increases, with lowering of the head it decreases.

This influence of the position of the head, caused by the tonic neck-reflexes of MAGNUS and DE KLEYN can promote the adaptation of the muscle-tonus and of the position of the limbs to the inclination of the floor. For if an animal stands on a board or table, which is moved up and down at the front or at the back, then the position of the head in space. is altered. Thereby the labyrinthine righting reflexes will begin to operate and the head will be brought back again to the original position in space.

If e.g. the top of the table is moved down, the labyrinthine righting reflexes will move the head backward, by which the supporting tonus of the hindlegs will decrease, that of the forelegs will increase. Consequently the former will flex, the latter will stretch. Labyrinth-stimuli are not indispensable to this adaptation, as labyrinthless dogs, also when they are blindfolded, also exhibit this altered position. It follows that other factors must be in operation. (Labyrinthless dogs, however, exhibit a proper adaptation only when the up-and-down movement of the table takes place slowly. For if the top of the table is moved up quickly, we shall see that on account of the absence of the semi-circular canal reactions the labyrinthless animals will turn a somersault backward and will be dashed to the ground.)

II. *The bending of the back.*

We have seen that touching the soles as well as applying a pressure on the soles influences the bending of the back and the tension of the dorsal muscles. Conversely also the bending of the back plays an influence on the supporting tonus of the legs. If the back is convex the supporting tonus of the hindlegs is strong ; if concave it is weak, and the hindlegs are twitching. This influence emanating from the back begins to operate with the adaptation to the static relations, if only the position of the bearing surface of the hindlegs changes, while that of the forelegs remains constant.

Also with men the bending of the back has a similar influence. Every one who happens to have been engaged in a wrestling-match, knows that as soon as he has succeeded in slightly twitching the back in the loins, it is an easy thing to flex the legs completely.

III. *The influence of the position of the forelegs on the supporting tonus of the hindlegs.*

If a dog is placed with its forelegs on a steady bearing surface, e.g. on the edge of a table, while the pads of the hindlegs rest on the hand, and if the trunk is moved backward, so that the forelegs are moved forward in the shoulders, we shall feel that the hindlegs are being stretched immediately and push the hand back with great force. This stretching attended with a strong resistance against passive flexion persists so long as the position of the animal is not altered. If, on the contrary, the trunk is moved forward, by which the forelegs go back in the shoulders, we shall feel the resistance decrease, and ultimately the hindlegs are twitched.

With dorsal position the same occurs. If you place one hand on the pads of the forelegs of the supine animal, and the other hand on the pads of the hindlegs, you'll also observe that with passive forward movement of the forelegs a stretching and an increase of the supporting tonus of the hindlegs occurs, and with a backward movement of the forelegs a decrease of the supporting tonus.

The significance of this influence for the adaptation to the static relations in standing on a bearing surface, that is moved up-and-down at the forepart or at the back, is self evident.

IV. *The influence of the position of the contralateral leg.*

If a dog is placed with one of his legs, say the right foreleg, on a table, while the contralateral leg, in casu the left one, rests on the hand, we feel that with sideway to-and-fro movement of the trunk the supported leg is flexed and stretched. With a movement of the trunk towards the left a stretching of the supported left foreleg occurs together with strong resistance against passive flexion, with a movement of the trunk to the right disappearance of the resistance and flexion of the leg. On closer investigation it appeared that this altered tonus was caused by the ab- and adduction of the right foreleg. For, if instead of the trunk the board on which the right foreleg rested, was moved to and fro, the reactions of the left foreleg occurred all the same. Also when the dog stands on one *hind*leg sideway to and fro movement of the trunk causes the other hindleg to exhibit alternately flexing and stretching movements.

A movement of the trunk antero-posteriorly yields similar reactions. If e.g. the animal stands on the right foreleg and if the trunk is moved forward, a forward stretching and likewise a strong resistance against passive flexion of the supported left foreleg will take place; when the trunk is moved in the opposite direction again, the resistance will disappear again and flexion of this leg will follow.

Also in the animal in dorsal position similar reactions can be elicited.

All the reactions discussed sub I—IV are tonic. They occur in normal, as well as in thalamus animals. In decerebellated dogs they appear very automatically, and cause excessively marked changes of the supporting tonus.

V. *The influence of the position of the leg in shoulder, resp. hip-joint on the supporting tonus of this leg.*

If we place a dog with one leg, say the right foreleg, on a table and place this leg in strong abduction, it will show but little supporting tonus; the muscles of fore- and upper-arm feel flaccid, the elbow is partly flexed and the leg yields even to very light pressure. If now we move the trunk dextrad, so that the leg is charged more heavily, and the markedly stretched adductors relax at the same time that the abductors become stretched, then the elbow and the shoulder will be seen to extend. The muscles of the leg now feel compact and pressure on the shoulder meets with strong resistance. The nearer the leg approaches the median position, the stronger the resistance of the leg will be, the stronger the supporting tonus will become.

Similar reactions will occur when the bearing surface of a straddle-legged animal is moved down at one of the sides, say, at the right side. Gravitation will then draw the trunk to the right and try to adduct the right legs.

If a leg, say the right-foreleg, stands in the median position, and the trunk is moved dextrad, while the toes are fixed with one hand on the bearing surface, so that the leg is adducted, the leg will twitch at once under the weight of the trunk, and all the supporting tonus disappears. This disappearance of the supporting tonus is part of a correcting-movement, for if the toes are not fixed on the table and the trunk is moved dextrad, it will be seen that at the moment the supporting tonus disappears the leg is raised, subsequently abducted, and placed more outwardly. This reaction is probably caused by the intense stretching of the abductors, and is very likely a proprioceptive correcting-movement. Quite conformable correcting movements occur, when by removing the trunk the leg is transposed from the median position in the shoulder, resp. hip, posteriorly, anteriorly or exteriorly. Besides for the adaptation to the static relations and to the recovery of the body-equilibrium, these reactions are important also in other respects. For these reactions explain the puzzling phenomenon in thalamus-animals that, although they lack the placing-reflexes and correcting-movements on stimuli emanating from the body-surface, they always place their legs in the proper way in standing, walking and falling. These reactions are lacking in decerebrated animals. In thalamus-animals they are as a rule always present, but sometimes they occur somewhat late. They are often absent in decerebellated animals shortly after the extirpation; afterwards they always come back but display typical deviations. First of all they appear too late i.e. only when as with normal dogs, the leg is transposed further posteriorly, interiorly or exteriorly, and secondly the movement is too strong, too large, the leg is raised too much, is put down too far and too forcibly and with too loud a rap.

If we place the decerebellated animal with one foreleg at one end of a board and move the trunk anteriorly, the animal will halt along to the other end with fewer paces than a normal dog.

If an animal stands on a board, that is being moved down at the one end, gravitation will pull the animal forward. The animal then adapts itself to the static relations, and shows resistance by stretching the forelegs, while the hindlegs flex. If you pull a thalamus dog standing on a horizontal board, forward, you'll also see, no matter whether you pull at its head, neck, chest- or back-skin, that the forelegs are stretched forward, and that the hindlegs are flexed.

By pulling the animal forward first of all a heavier weight presses upon the forelegs and secondly the flexors of the metacarpophalangeal joint and the wrist, and also the extensors of elbow and shoulder are passively stretched. If the animal stands on a board that is moved down on the rightside, gravitation will draw the animal dextrad. It adapts itself by resisting outwardly, i.e. by stretching and abduction of the right legs, while the left legs will flex. Quite the same reactions of the legs appear when a normal, thalamus-, or decerebellated dog standing on a horizontal board, is pulled or pushed by the skin to the right. This pulling causes the right legs to be more heavily charged and the abductors of the right shoulder and hip are passively stretched.

The appearance of these reactions depends only on the direction of the force, not on the point of application.

Still, the latter must have some influence, as is evident from thalamus dogs. If you take a thalamus dog by the head without pulling, the animal will try to get free by pulling backwards; if, however, you take him by the tail the animal will get free by pulling forwards. If you take him by the right ear, the animal will turn his head to the left or walks round in circles to the left, right against the hands of the clock. If, on the contrary you hang a clamp at the skin of chest or hip of the right side, the animal will walk to the right.

All the above-described reactions, coming into play with standing and with adaptation to the charging and to the static relations, disappear after severing the posterior-roots, while the reactions on exteroceptive stimuli, such as magnet-reaction, disappear already after cutting the cutaneous nerves.

Regarding the localization of the centra of these reactions we saw already:

I. That after transverse section of the spinal cord most of these reactions are lacking, whereas others are considerably disturbed (touching the sole sometimes gives a brief stretching, pressure on the sole a brief fixation), so that standing is out of the question.

II. That also after decerebration most of these reactions are absent, while the others exhibit considerable disturbances also now.

III. That in thalamus-animals all the reactions are present, except the proper placing of the legs on optic stimuli and on stimuli emanating from the body-surface and also excepting the magnet-reactions.

IV. That after unilateral extirpation of the cerebrum the placing-reflexes on stimuli emanating from the body-surface, are absent at the legs on the side of the intact half of the cerebrum, that the other reactions, also the magnet-reactions occur, as well at the homolateral as at the contralateral legs.

V. That in decerebellated animals all the reactions are present, most of them even excessively. (This holds good only when the animals have accommodated themselves as well as possible, i.e. 3—6 months after the extirpation. On the first days after the extirpation most of these reactions do not appear and after this they are still disturbed for some time.)

In decerebellated animals some of these reactions are merely exclusively excessive, hypermetric; so e.g. the hypermetrical stretching and fixation of the extremities and the excessive bending of the back on touching the soles. Nothing was noted of too long a latency. With other reactions, on the contrary, as with proprioceptive correcting movements the lateness as well as excessiveness was observed. That these disturbances of the static reactions will contribute to the appearance of the so-called cerebellar ataxy, is evident.

In man standing is disturbed or quite impossible in cases of serious tabes (affections of the posterior roots and horns), of serious poliomyelitis

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	Spinal animal	Decerebrated animal	Thalamus-animal	Decerebellated animal
Placing-reflexes	—	—	—	+
Magnet-reaction	—	—	—?	+, excessive
Pos. supporting reaction	disturbed	disturbed	+	+ „
Neg. supporting reaction	—	disturbed	+	+ „
"Wiebel reactions" [1])	—	disturbed	+	+ „
Proprioceptive correcting movements	—	—	+	+ „
Other adaptation reactions	—	—	+	+ „
Neck righting reflexes	—	+	+	+
Other „ reflexes	—	—	+	+

(affections of the anterior horns) of serious neuritis caused by lues, beriberi etc., and in cases of muscular dystrophia, so in general with interruption of the spinal reflex arc. Then again with transverse lesions of the spinal cord, quite in keeping with the observations on animals. RIDDOCH and LHERMITTE observed in wounded soldiers also that the other phenomena after total transverse lesions of the spinal cord in man resemble completely those displayed by animals, while also after transverse lesions of the mesencephalon decerebrate rigidity occurs in man with a similar symptomatic aspect. With these, and also with other affections characterized by strong rigidity, as e.g. with serious spastic spinalparalysis, standing is disturbed in man. Standing is further impossible in man in diseases with still unknown localisations, as in cases of chorea mollis, athetose double, and others, finally also in nurslings.

The clinical examination of the muscle tonus is generally performed in dorsal position of the patient. Therefore, also the supporting tonus in animals was examined while the animal was lying on its back. It then appeared, that with normal animals in dorsal position pressure on the soles yielded far less supporting tonus than with the standing animal. This difference was much less in decerebellated animals. The following observations are especially remarkable. They were made in a dog of which the right half of the cerebellum had been extirpated, in the stadium in which the animal was able to stand again unsupported.

We see from following table:

firstly: that when the animal stands on one of the hindlegs the hindleg on the side of the extirpation had a weaker supporting tonus;

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[1]) Reactions emanating from the contralateral limb.

Dog Fox ($6^{1}/_{2}$ kg.)
1 month after extirpation of the *right* half of the cerebellum.

Strength of the supporting tonus in kilogrammes

	Right hindleg		Left hindleg	
Standing on one leg	$+ 6^{1}/_{2}$	$- 7^{1}/_{2}$	$+ 8$	$- 9$
In dorsal position	$+ 7$	$- 7^{1}/_{2}$	$+ 2^{1}/_{2}$	$- 3$

secondly : that the supporting tonus of this leg was as strong with dorsal position as with standing ;

thirdly : that the supporting tonus of the contralateral leg was much weaker in dorsal position than in standing, so that with dorsal position the supporting tonus of the homolateral leg was stronger than that of the contralateral one.

On closer inspection we observed that this decrease of the supporting tonus of the contralateral leg was caused by the pressure on the back. This pressure mostly produces a distinct decrease in normal dogs, much less in decerebellated dogs, whereas in decerebrated animals the rigidity often increases in dorsal position under the influence of labyrinthine stimuli.

Also by other stimuli on the back the supporting tonus decreases as, e.g. taking hold of the dog with two hands by the skin of his back. This decrease also occurs with thalamusdogs.

If a patient lying in bed on his back is examined, the abdominal wall will tighten when palpation takes place. The patient is then told to keep calm, not to obstruct, so that then the tightening most often ceases. The reflex of the abdominal wall is then inhibited voluntarily, psychically. Quite similar phenomena are observable when investigating the tonus of the muscles of the extrimities, as well in man as in animal.

If you speak gruffly to a dog that is being examined, the supporting tonus often disappears directly and neither touching nor pressure is of any influence. Every one knows that a dog's legs twitch abruptly when you speak to it gruffly.

CONCLUSIONS.

I. Normal standing is brought about by a number of extero-, and proprioceptive reactions and the centra of these reactions are situated in different parts of the central nervous system.

II. All these reactions occur (most of them excessively) in decerebellated animals, and probably part of the aspect of the "cerebellar ataxy" depends on the excessiveness of these reactions.

III. Tonus and supporting tonus should be sharply differentiated, as in man and in animal hypertonia is often attended with a decreased supporting tonus; e.g. the hemiplegic leg shows most often less supporting tonus than the healthy one.

IV. Touching of the sole is alone competent to alter the muscle-tonus of the whole body of an animal, a precise investigation of the muscle-tonus is, therefore, very difficult, and the description of this investigation should always be accompanied by a minute report of the circumstances under which it took place.

V. In the clinic the examination of the muscle-tonus occurs mostly with dorsal position of the patient. In animals this position (apart from the occurrence of labyrinthine stimuli) inhibits certain reflexes that increase the muscle tonus.

Utrecht. *From the Pharmacological Laboratory.*

Anatomy. — *The Connection of the Cortex with the Neostriatum and Palaeostriatum in rabbits.* By Dr. L. COENEN. (Communicated by Dr. C. U. ARIËNS KAPPERS and Prof. B. BROUWER).

(Communicated at the meeting of September 24, 1927).

The question if there are tracts, which connect the cortex and neostriatum (putamen and nucleus caudatus) or palaeostriatum (globus pallidus) has again recently stimulated many neurologists. Formerly it was generally believed, that a fibre-connection between cortex and striatum did not exist. In 1872 VON GUDEN mentioned as a result of his experiments that cortical lesions have no influence upon the corpus striatum. Also FLECHSIG (1876) and later WERNICKE emphasised that the corpus striatum is independent of the cortex and has no connections with it.

KINNIER WILSON (1914) after extensive investigation of experimental lesions of the neostriatum on monkeys found no degenerated fibres to the cortex, and concluded that a connection between striatum and cortex does not exist. The few degenerated fibres that he found he considered to be fibres which only passed the nucleus caudatus to reach the capsula interna.

As late as 1919 C. and O. VOGT deny the existance of connections between cortex and striatum (nucleus caudatus and nucleus lentiformis). They regard the fibres which seem to establish such connections as fibres which pass the striatum to join the capsula interna.

There were, however, some neurologists who believed such connections to exist. Among these were BIANCHI and D'ABUNDO (1886) who had removed the cortex from a dog and had found a reduction of volume of the globus pallidus and proliferation of the neuroglia in the nucleus caudatus and lentiformis; and MARINESCU who in 1895 after removal of the lobus frontalis, found with the Marchi method degenerated fibres running from this lobe along the frontal segment of the capsula interna to the nucleus caudatus in which they disappeared. RAMON J. CAJAL (1895), though granting that there were no direct fibres from the cortex to the striatum, found in rodentia (rat and mouse) that the fibres from the pyramidal cells of the cortex, which run to the capsula interna, send out collaterals to the striatum. DEJERINE, LONG and DEJERINE (1898) believed, that in cortical lesions in man no degenerated fibres could be demonstrated in the nucleus caudatus and the putamen but found in cortical and subcortical lesions a slight degeneration in the globus pallidus. In 1918 ECONOMO described as the result of a primary lesion of the corpus striatum degenerated fibres running to the cortex.

Especially however VON MONAKOW and MINKOWSKY gave us a clearer formulation of these vague conceptions.

VON MONAKOW (1925) found after extirpation of the cortex in a dog a distinct atrophy of the corpus striatum. His collaborator MINKOWSKY removed several parts of the cortex in monkeys and got the following results:

From the gyrus praerolandus fibres run to the nucleus caudatus and from the lobus frontalis to the nucleus caudatus as well as to the globus pallidus. He also confirmed the presence of pallido-frontal fibres, which had first been observed by VON MONAKOW.

Recently KODAMA studying the development of the globus pallidus and corpus striatum in embryos found that the striatum subcallosum establishes an association between cortex and nucleus caudatus; and that in a fetus of 6 months myelinated fibres occur in the globus pallidus (oral part) which pass in the capsula interna and partly disappear in the corona radiata. KODAMA does not believe, that these are simply fibres which pass the striatum because they are caudally no where visible in the peduncules. He believes that they disappear in the globus pallidus.

For my experiments I used rabbits. The left side of the skull was opened and with a sharp spoon a part of the cortex was removed, a different area in each rabbit. As rigid asepsis as possible was maintained so that no suppuration should affect the site of the lesion and thus cause complications in the degeneration. The rabbits were allowed to live 18 days, when they were hanged and the brains treated after the Marchi method.

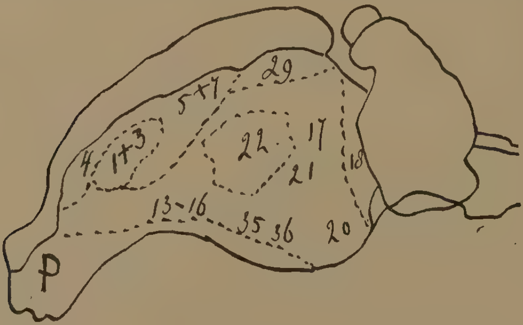

Fig. 1.

In this way ten rabbits were operated and the following fields (after the map of WINKLER and POTTER) were removed. Fig. 1:

1. field 4, 1 + 3 (area prae- and postcentralis).
2. field 4, 1 + 3, 5 + 7 (area prae- and postcentralis and parietalis).
3. field 4, 1 + 3, 5 + 7 (area prae- and postcentralis and parietalis).

4. field 4, 1 + 3. 5 + 7,22 (area prae- and postcentralis, parietalis, temporalis superior).
5. field 4, 1 + 3, 5 + 7,17[1], 18[1], 29 (area prae- and postcentralis, parietalis, striata, occipitalis, retrosplenialis).
6. field 1 + 3, 5 + 7, (area postcentralis, and parietalis).
7. field 1 + 3, 5 + 7, 22, (area occipitalis, parietalis, temporalis superior).
8. field 17, 21, 22 (area occipitalis, temporalis media and superior).
9. field 20, 21, 22 (area temporalis inferior, media superior).
10. field 20, 21 (area temporalis inferior and media).

Of these series I shall treat only the degenerations in the principal ones and after that give my conclusions.

1. *Serie 1. B. 205.* Removed field 4, 1 + 3 (area prea- and postcentralis).

The intention was to remove the left frontal pole. At the operation it was first thought that we were dealing with an anomaly of the left skull, and we therefore passed over to the right skull. Here, however, the same condition was found, so that now again the left side was further operated upon. Hence by this operation we cut the left as well as the right frontal pole. There was slight bleeding. The rabbit was killed after 18 days. The left as well as the right pole appeared to have been cut. As a result of the bleeding there was much infiltration at both poles.

It may be stated that field 4 (area praecentralis, field 1 + 3 in part, (area postcentralis), and a very small piece of 5 + 7 (area parietalis) (according to the map of Winkler and Potter) had been removed. Fig. 2 gives a sketch of the lesion. In section 18 we observed the corpus

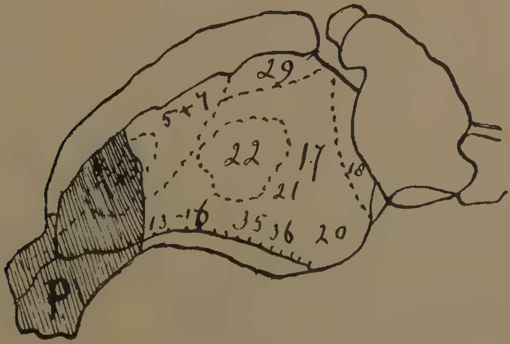

Fig. 2.

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[1] I believe it better to call area 17 "area striata" and area 18 "area occipitalis", after Brodmann.

callosum on both sides full of degenerative granulae, more on the right side than on the left.

The centrum semiovale as well was more involved by this process on the right than on the left. Further, over the dorsal part of the nucleus caudatus a long line of degenerative granulae runs which inclines slightly to the side of the ventricular surface of the striatum (capsula intima PŘECECHTĚL.[1]) From this line on the right side long lines of degenerative granulae were observed to radiate (fig. 3) which were not visible on the left side.

The line at the dorsal side of the nucleus caudatus was also much smaller in extent on this side.

Section 24. As well on the left side as on the right many degenerative granulae are seen at the side of the capsula interna which borders the nucleus caudatus, These form a long line of small islands, where the granulae are most numerous and whence strips of degenerative granules run to the nucleus caudatus (fig. 4).

Serie 4 B. 176. Removed field 4 in part (area praecentralis), 1 + 3 in part (area postcentralis) 5 + 7 in part (area parietalis) and 22 nearly entirely (area temporalis superior). A sketch of the *lesion* in fig. 5.

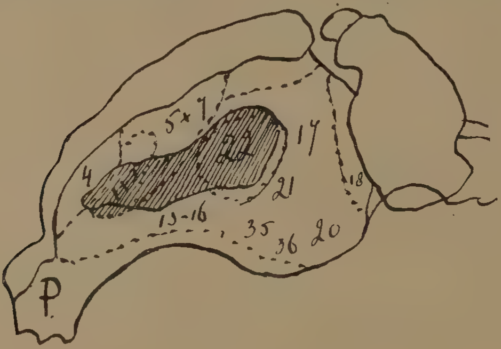

Fig. 5.

Section 12. The corpus callosum on the right side is full of degenerative granules; on the left side there are none at all. Here are also many in the capsula interna on the right. Again over the dorsal side of the nucleus caudatus (capsula intima) a long strip of degenerative granulae is seen to run from the beginning of the capsula interna (fig. 6 and 7).

Section 16. Here also a degenerative tract is seen from the centrum semiovale and the fasciculus subcallosus along the dorsal edge of the

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[1]) The fibrelayer, which covers the corpus striatum (specially the nucleus caudatus) on the ventri-cular side. This capsula intima is specially strong in the Elephant where it was first described by PŘECECHTĚL (1925).

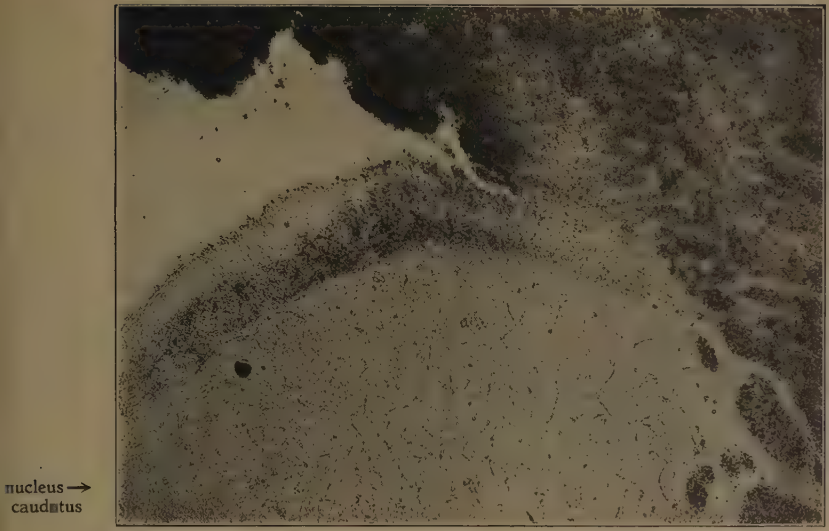

Fig. 3. Degeneration in the capsula intima. Serie I. B. 205. (section 18).

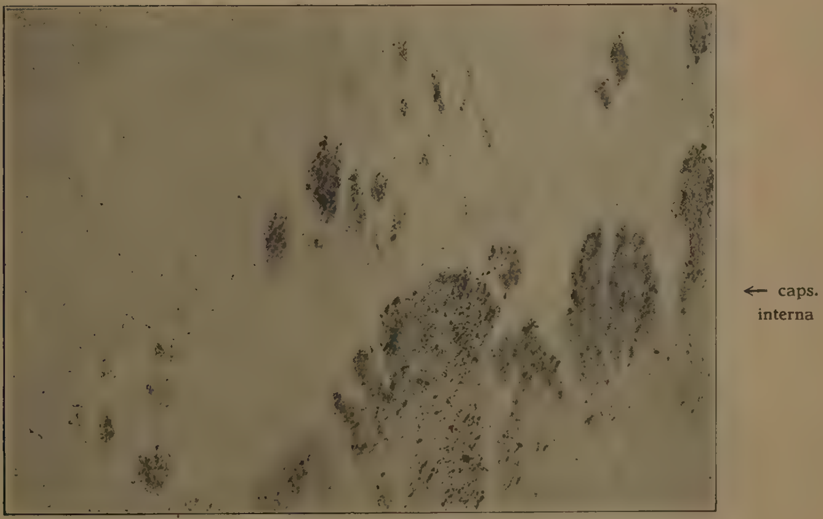

Fig. 4. Degenerated granules from the capsula interna to the nucleus caudatus. Serie I. B. 205 (section 24).

nucleus caudatus (fig. 6 and 7). At the dorsal side of the capsula interna one sees again an edge of degenerative granules, interrupted here and there. From here degenerative granules radiate to the centre of the nucleus caudatus (fig. 6 and 8). Further on from the capsula interna tracts filled with degenerative granulae run to the centre of the globus pallidus (fig. 9).

Section 17. N°. 8. Fig. 6 and 7 show a tract filled with granules, which runs over the dorsal ventricular lining (capsula intima) of the nucleus caudatus. Further on the capsula interna shows at its dorsal side a darker part, filled with granules, here and there interrupted, whence degenerated tracts radiate into the nucleus caudatus (fig. 8).

One observes as well rows of granules, running from the capsula interna to the globus pallidus (fig. 9). Fig. 7, 8, 9 show some of these degenerations enlarged. Fig. 7 shows the tract, which runs from the fasciculus subcallosus in the dorsal lining of the nucleus caudatus, and whence numerous degenerated strips continually run into the above named nucleus.

Fig. 8. Shows strips of degenerated tracts running from the dorsal part of the capsula interna into the nucleus caudatus and finally fig. 9 shows the tracts which run from the capsula interna into the globus pallidus.

Serie IX. B. 209. Removed field 22 (temporalis superior) 21 (temporalis media) 20 (part of temporalis inferior). Fig. 10 shows the lesion.

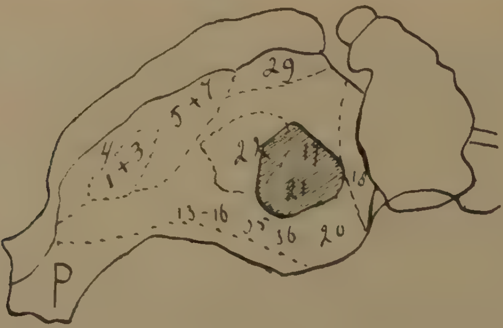

Fig 10.

Section 21. 1. Fibres at the dorsal side of the capsula interna running in the nucleus caudatus (fig. 11).

Section 23. Fibres from the capsula interna to the globus pallidus (fig. 12).

Resume of personal investigations.

Serie 1. B. 205. Removed area prae- and postcentralis.

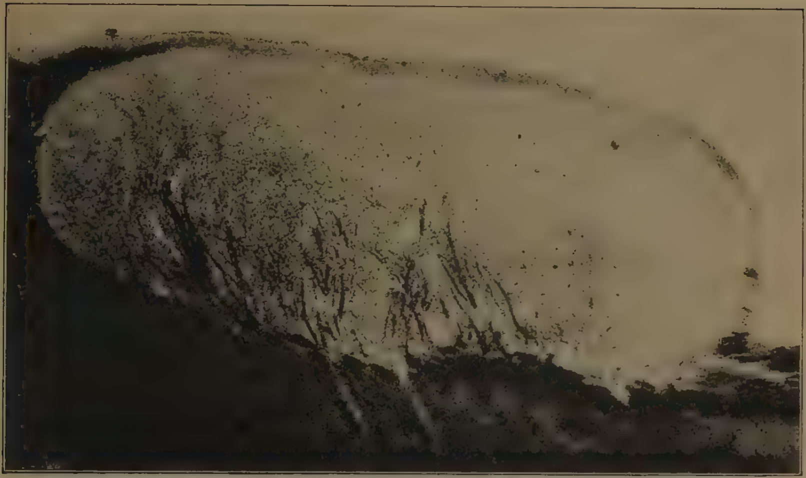

Fig. 6. Degenerations on the dorsal side of the capsula interna.
Serie IV. B. 176.

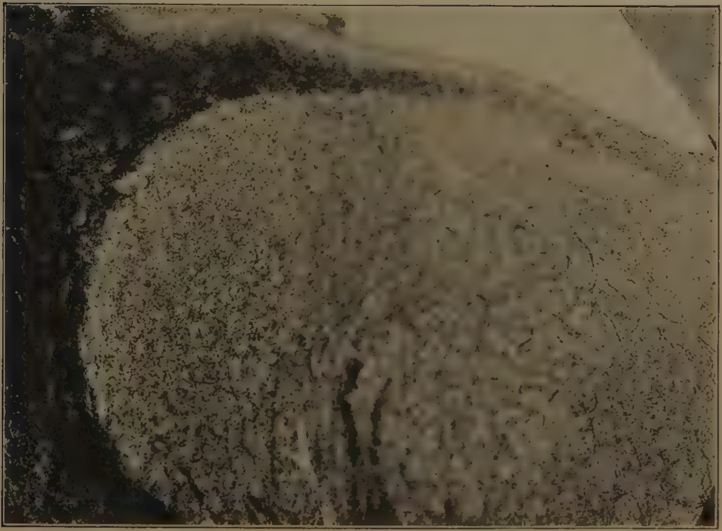

Fig. 7. Degeneration in the capsula intima. Serie IV. B. 176.

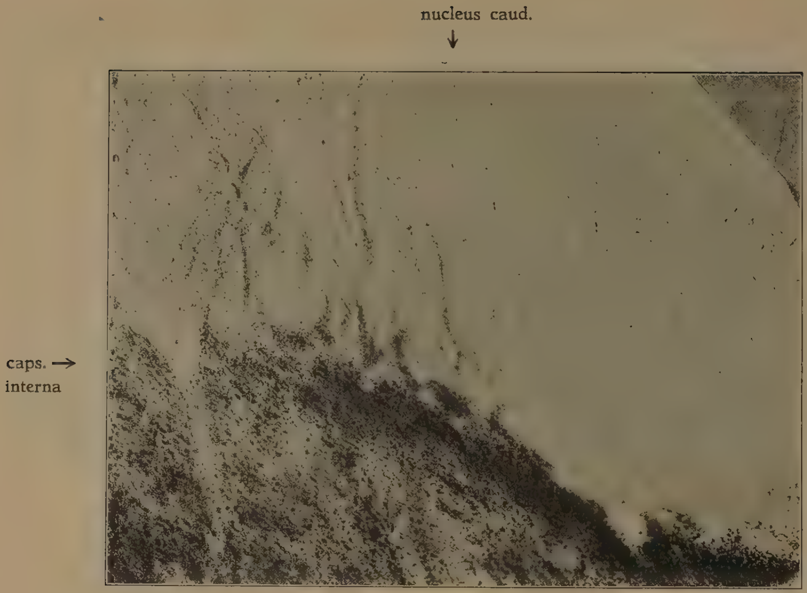

Fig. 8. Degenerated granules from the capsula interna to the nucleus caudatus. Serie IV. B. 176 (section 17).

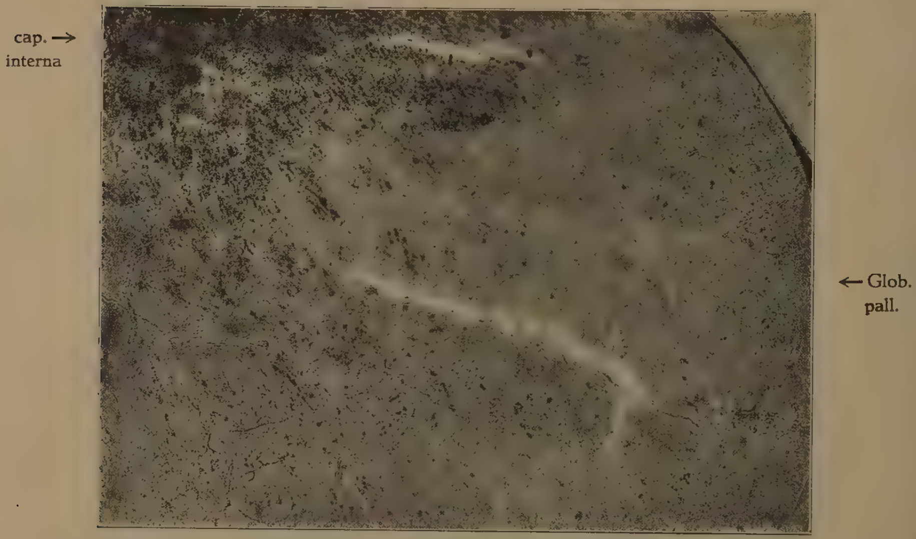

Fig. 9. Degenerated granules from the capsula interna to the palaeostriatum. Serie IV. B. 176 (section 17).

Result: 1. Degenerated fibres from the fasciculus subcallosus run dorsally over the nucleus caudatus and pass into it.

2. Degenerated fibres spread from the dorsal side of the capsula interna into the nucleus caudatus.

Serie IV. B. 176. Removed area prae- and postcentralis (both partially) area parietalis (in part), and area temporalis superior (nearly the whole of this area).

Result: 1. Degenerated fibres from the fasciculus subcallosus run dorsally over the nucleus caudatus and pass into it.

2. Degenerated fibres from the dorsal part of the capsula interna spread in the nucleus caudatus.

3. Degenerated fibres from the capsula interna run into the globus pallidus.

Serie IX. B. 209. Removed field 22 (temporalis superior), 21 (temporalis medius), 20 (temporalis inferior).

Result: 1. Degenerated fibres at the dorsal side of the capsula interna pass over into the nucleus caudatus.

2. Degenerated fibres passing from the capsula interna into the globus pallidus.

In none of these series degenerated fibres can be seen in the *putamen*. There is no direct connection between the cortex and putamen. I found degenerations in the following series as well:

In series V, where field 4 (area praecentralis), 1 + 3 (area postcentralis), 5 + 7 (area parietalis), 29 (area retrosplenialis), 17 (area striata), 18 (area occipitalis) have been removed, degenerated fibres at the dorsal side of the nucleus caudatus are distinctly visible, penetrating into the nucleus caudatus.

In series VIII, where 17 (area striata), 21 (temporalis media) and 22 (temporalis superior) and in series X, where the fields 20 (temporalis inferior) and 21 (temporalis media) have been removed, a distinctly degenerated fasciculus subcallosus and a degeneration in the nucleus caudatus could be observed energing from the dorsal edge of the capsula.

CONCLUSIONS.

In rabbits a connection exists between the cortex and the nucleus caudatus and globus pallidus coming from the following parts of the mantle:

1. There is a connection between the area centralis and temporalis and nucleus caudatus by fibres which run through the fasciculus subcallosus partly under the dorsal ventricular lining of the nucleus caudatus and partly along the dorsal edge of the capsula interna, passing into the anterior part of the nucleus caudatus.

2. There is a connection of the area temporalis with the globus

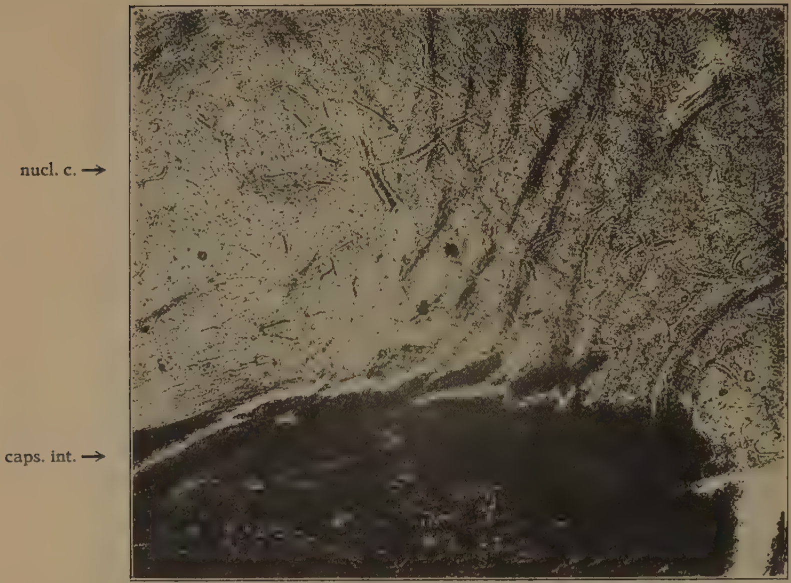

Fig. 11. Degenerated granules from the capsula interna to the corpus striatum. Serie IX. B. 209.

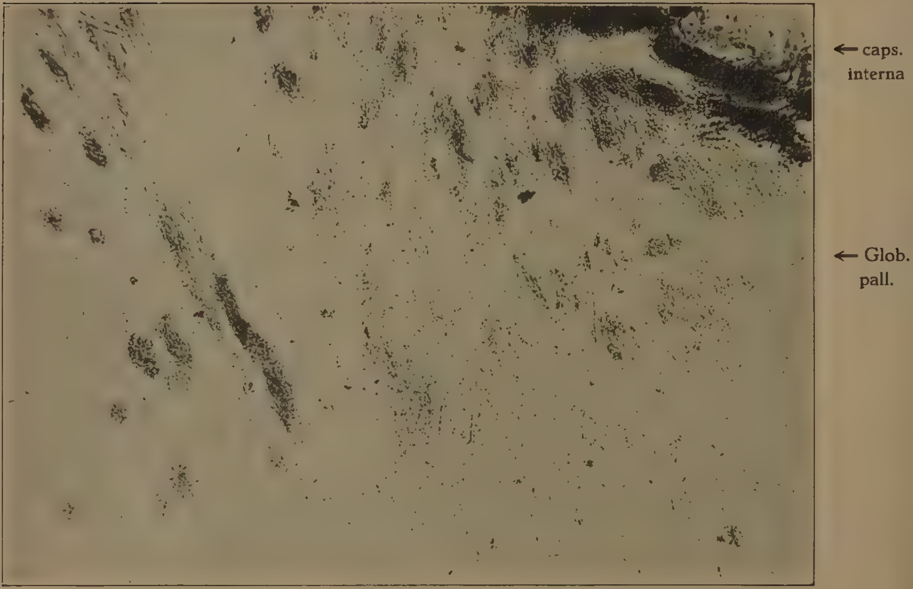

Fig. 12. Degeneration of the capsula interna to the palaeostriatum. Serie IX. B. 209.

pallidus by fibres, which run from the area temporalis into the capsula interna and from here into the globus pallidus (palaeostriatum) (KAPPERS).

3. There is no cortical connection with the putamen.

Haarlem, October 1927.

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Mathematics. — *Method of Calculating the mean Errors of the Standard Deviations, Coefficients of Correlation, and Coefficients of Regression, with n linearly correlated Variables.* By Prof. M. J. VAN UVEN. (Communicated by Prof. A. A. NIJLAND.)

(Communicated at the meeting of September 24, 1927).

For determining the mean errors and the mutual correlative relations of several characteristic correlation parameters (standard deviations, coefficients of correlation, coefficients of regression etc.). PEARSON en FILON [1]) gave a method long ago.

The present paper, treating only of linear correlation, follows — in our opinion — a shorter way, gives moreover in the general case of n variables the mean errors of all the most important characteristic correlation parameters, and furnishes at the same time all the data necessary for computing the mutual correlative relations between these parameters.

In treating the problem of the correlation of n variables n quantitative characteristics $x_1\, x_2, \dots x_n$ are considered, occurring with each of a great number (N) of individuals. Each individual gives a set of values $x_1, x_2, \dots x_n$. All the individuals together give for x_1 the mean $\frac{[x_1]}{N} = X_1$, for x_2 the mean $\frac{[x_2]}{N} = X_2$, generally for x_k the mean $\frac{[x_k]}{N} = X_k$ ($k = 1, 2, \dots n$); the symbol [ ] denotes a summation over the N individuals.

According to the theory of linear correlation, the set of the means $X_1, X_2, \dots X_k, \dots X_n$ is the most probable one.

Each individual value x_k deviates from its mean X_k by the amount

$$u_k = x_k - X_k = x_k - \frac{[x_k]}{N};$$

therefore each individual can also be characterised by its set $u_1, u_2, \dots u_k, \dots u_n$. The correlation being supposed to be linear, the law of distribution of these sets among the different individuals is given by the differential probability formula

$$dW = K \, . \, e^{-f(u_1, u_2, \dots u_n)} \, du_1 \, du_2 \dots du_n \, ,$$

where $f(u_1, u_2, \dots, u_n)$ is a homogeneous definite quadratic form in the magnitudes u_k. K being a constant, the value of which can be determined by the condition

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[1]) K. PEARSON and L. N. G. FILON: Mathematical Contributions to the Theory of Evolution: IV. On the Probable Errors of Frequency Constants, and on the Influence of Random Selection on Variation and Correlation. Phil. Trans. series A, Vol. **191**, (1898), p. 229.

$$\int\limits_{u_1=-\infty}^{+\infty}\cdots\int\limits_{u_k=-\infty}^{+\infty}\cdots\int\limits_{u_n=-\infty}^{+\infty} dW = 1 \quad . \; . \; . \; . \; . \; . \; , \qquad (1)$$

Writing $f(u_1, u_2, \ldots, u_n)$ in the form

$$\left.\begin{aligned} f &\equiv \tfrac{1}{2}(p_{11}\, u_1^2 + 2p_{12}\, u_1\, u_2 + \ldots + 2p_{kl}\, u_k\, u_l + \ldots + p_{nn}\, u_n^2) \\ &\equiv \tfrac{1}{2}\sum_{\alpha=1}^{n}\sum_{\beta=1}^{n} p_{\alpha\beta}\, u_\alpha\, u_\beta\,, \end{aligned}\right\} \; . \qquad (2)$$

and putting

$$P = |p_{kl}| = \begin{vmatrix} p_{11}, & \ldots, & p_{1n} \\ p_{12}, & \ldots, & p_{2n} \\ \vdots & & \vdots \\ p_{1n}, & \ldots, & p_{nn} \end{vmatrix},$$

K appears as

$$K = \sqrt{\frac{P}{(2\pi)^n}}\,;$$

whence

$$dW = \frac{\sqrt{P}}{(2\pi)^{\frac{n}{2}}}\, e^{-f}\, du_1 \ldots du_n \quad . \; . \; . \; . \; . \; . \; . \qquad (3)$$

So the probability formula contains n magnitudes p_{kk} and $\frac{n(n-1)}{2}$ magnitudes p_{kl}, the values of which must be determined empirically, i.e. be deduced from the observations.

They are determined in the simplest way by calculating the quadratic means

$$m_{kk} = \frac{[u_k^2]}{N} \text{ and } m_{kl} = \frac{[u_k\, u_l]}{N}\,.$$

By equalizing the theoretical expressions for m_{kk} and m_{kl} to the values empirically found, we obtain $n + \frac{n(n-1)}{2} = \frac{n(n+1)}{2}$ equations for determining the $\frac{n(n+1)}{2}$ magnitudes p_{kk}, p_{kl}.

The theoretical expressions for m_{kk} and m_{kl} run

$$m_{kk} = \int\limits_{u_1=-\infty}^{+\infty}\cdots\int\limits_{u_n=-\infty}^{+\infty} u_k^2\, dW \quad , \quad m_{kl} = \int\limits_{u_1=-\infty}^{+\infty}\cdots\int\limits_{u_n=-\infty}^{+\infty} u_k\, u_l\, dW.$$

Putting

$$\frac{1}{(2\pi)^{\frac{n}{2}}}\int\limits_{u_1=-\infty}^{+\infty}\cdots\int\limits_{u_n=-\infty}^{+\infty} e^{-f}\, du_1 \ldots du_n = I,$$

we may write, by means of (3), the equation (1) in the form

$$\sqrt{P}\,.\,I=1$$

or

$$I=P^{-1/2}.$$

Differentiating both members of this equation with respect to p_{kl}, we find

$$\frac{\partial I}{\partial p_{kl}}=-\tfrac{1}{2}\,P^{-3/2}\,\frac{\partial P}{\partial p_{kl}}\,. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

Now we have (because $\frac{\partial f}{\partial p_{kl}}=u_k\,u_l$)

$$\frac{\partial I}{\partial p_{kl}}=-\frac{1}{(2\pi)^{\frac{n}{2}}}\int\limits_{u_1=-\infty}^{+\infty}\cdots\int\limits_{u_n=-\infty}^{+\infty}u_k\,u_l\,e^{-f}\,du_1\ldots du_n=-\frac{1}{\sqrt{P}}\,.\,m_{kl}\,.$$

In differentiating the symmetrical determinant P it would be advisable to consider it as resulting from the non-symmetrical determinant P' (in which $p_{lk}\neq p_{kl}$).

Representing the subdeterminant, or minor, of p_{kl} in the determinant P (resp. P') by P_{kl} (resp. P'_{kl}), we have

$$\frac{\partial P}{\partial p_{kl}}=\frac{\partial P'}{\partial p_{kl}}+\frac{\partial P'}{\partial p_{lk}}=P'_{kl}+P'_{lk}=2P_{kl}\,.$$

Hence from (4) ensues

$$-\frac{1}{\sqrt{P}}\,.\,m_{kl}=-\tfrac{1}{2}\,P^{-3/2}\,.\,2\,P_{kl}\,,$$

or

$$m_{kl}=\frac{P_{kl}}{P}\,.$$

Differentiating with respect to a coefficient p_{kk} with equal subscripts, we find $\left(\text{in virtue of } \frac{\partial f}{\partial p_{kk}}=\frac{1}{2}\,u_k^2\right)$

$$\frac{\partial I}{\partial p_{kk}}=-\tfrac{1}{2}\,.\,\frac{1}{(2\pi)^{\frac{n}{2}}}\int\limits_{u_1=-\infty}^{+\infty}\cdots\int\limits_{u_n=-\infty}^{+\infty}u_k^2\,e^{-f}\,du_1\ldots du_n=-\frac{1}{2\sqrt{P}}\,.\,m_{kk},$$

and

$$\frac{\partial P}{\partial p_{kk}}=\frac{\partial P'}{\partial p_{kk}}=P'_{kk}=1\,.\,P_{kk},$$

whence

$$m_{kk}=\frac{P_{kk}}{P}\,.$$

So the equation

$$m_{kl} = \frac{P_{kl}}{P} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{I}^a)$$

holds equally good in the case $l = k$.

Now

$$\sum_{\alpha=1}^{n} P_{i\alpha}\, p_{i\alpha} = P \quad , \quad \sum_{\alpha=1}^{n} P_{i\alpha}\, p_{j\alpha} = 0 \qquad \text{for } j \neq i,$$

or, introducing the symbol ω_{ij} defined by

$$\omega_{ii} = 1 \quad , \quad \omega_{ij} = 0 \qquad \text{for } j \neq i,$$

$$\sum_{\alpha=1}^{n} P_{i\alpha}\, p_{j\alpha} = \omega_{ij}\, P,$$

or, in consequence of (I^a),

$$\sum_{\alpha=1}^{n} m_{i\alpha}\, p_{j\alpha} = \omega_{ij} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Comparing this relation with

$$\sum_{\alpha=1}^{n} m_{i\alpha}\, M_{j\alpha} = \omega_{ij} \, . \, | m_{kl} | = \omega_{ij}\, M,$$

we find

$$p_{ij} = \frac{M_{ij}}{M}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{I}^b)$$

moreover we have

$$P = | p_{ij} | = \frac{| M_{ij} |}{M^n} = \frac{1}{M},$$

or

$$P \, . \, M = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{I}^c)$$

The equations (I^a), (I^b) and (I^c), well known indeed since long, give the relation between the magnitudes m_{kl} and the magnitudes p_{kl}. This relation is reciprocal.

Besides the quadratic means m_{kl}, the biquadratic mean

$$m_{ijkl} = \int_{u_1=-\infty}^{+\infty} \cdots \int_{u_n=-\infty}^{+\infty} u_i\, u_j\, u_k\, u_l \, dW$$

will also be calculated.

To this purpose we differentiate both members of (4) with respect to p_{ij}.

So we find

$$\frac{\partial^2 I}{\partial p_{kl}\, \partial p_{ij}} = \frac{-2P \dfrac{\partial^2 P}{\partial p_{kl}\, \partial p_{ij}} + 3 \dfrac{\partial P}{\partial p_{ij}} \dfrac{\partial P}{\partial p_{kl}}}{4 P^{5/2}}.$$

Now

$$\frac{\partial^2 I}{\partial p_{kl}\,\partial p_{ij}} = + \frac{1}{(2\pi)^{\frac{n}{2}}} \int\limits_{u_1=-\infty}^{+\infty} \cdots \int\limits_{u_n=-\infty}^{+\infty} u_i\, u_j\, u_k\, u_l\, e^{-f}\, du_1 \ldots du_n = \frac{m_{ijkl}}{\sqrt{P}}\,.$$

Further

$$\frac{\partial^2 P}{\partial p_{kl}\,\partial p_{ij}} = \frac{\partial}{\partial p_{ij}} \left(\frac{\partial P'}{\partial p_{kl}} + \frac{\partial P'}{\partial p_{lk}} \right) + \frac{\partial}{\partial p_{ji}} \left(\frac{\partial P'}{\partial p_{kl}} + \frac{\partial P'}{\partial p_{lk}} \right) =$$

$$= P'_{kl,\,ij} + P'_{lk,\,ij} + P'_{kl,\,ji} + P'_{lk,\,ji}\,.$$

$P'_{kl,\,ij}$ is the complementary minor of $\begin{vmatrix} p_{ij} & p_{il} \\ p_{kj} & p_{kl} \end{vmatrix}$; therefore

$$P'_{kl,\,ij} = \frac{P'_{ij}\, P'_{kl} - P'_{il}\, P'_{kj}}{P'} = \frac{P_{ij}\, P_{kl} - P_{il}\, P_{jk}}{P}\,.$$

Likewise:

$$P'_{lk,\,ij} = \frac{P'_{ij}\, P'_{lk} - P'_{ik}\, P'_{lj}}{P'} = \frac{P_{ij}\, P_{kl} - P_{ik}\, P_{jl}}{P},$$

$$P'_{kl,\,ji} = \frac{P'_{ji}\, P'_{kl} - P'_{jl}\, P'_{ki}}{P'} = \frac{P_{ij}\, P_{kl} - P_{ik}\, P_{jl}}{P},$$

$$P'_{lk,\,ji} = \frac{P'_{ji}\, P'_{lk} - P'_{jk}\, P'_{li}}{P'} = \frac{P_{ij}\, P_{kl} - P_{il}\, P_{jk}}{P}.$$

Hence

$$\frac{\partial^2 P}{\partial p_{kl}\,\partial p_{ij}} = \frac{4\, P_{ij}\, P_{kl} - 2\, P_{ik}\, P_{jl} - 2\, P_{il}\, P_{jk}}{P}$$

and

$$\frac{\partial^2 I}{\partial p_{kl}\,\partial p_{ij}} = \frac{m_{ij\,kl}}{\sqrt{P}} = \frac{-8\, P_{ij}\, P_{kl} + 4\, P_{ik}\, P_{jl} + 4\, P_{il}\, P_{jk} + 12\, P_{ij}\, P_{kl}}{4\, P^{5/2}}$$

$$= \frac{P_{ij}\, P_{kl} + P_{ik}\, P_{jl} + P_{il}\, P_{jk}}{P^2 \sqrt{P}}$$

therefore, by (I^a),

$$m_{ij\,kl} = m_{ij}\, m_{kl} + m_{ik}\, m_{jl} + m_{il}\, m_{jk} \quad \ldots\ldots \quad (6)$$

This formula holds equally good in the case that two or more of the subscripts i, j, k, l are equal.

If the actual set of observations is supposed to be repeated many times, the magnitude m_{kl} will generally take different values. Should the different sets of observations not diverge too much from each other, these values of m_{kl} will be rather closely grouped about their mean $\overline{m}_{kl}$. The number of repetitions increasing perpetually, the limit to which $\overline{m}_{kl}$ approaches may, with some reason, be called the "true" value of m_{kl}.

As, in reality, we have to operate with only one set of observations, this true value $\overline{m}_{kl}$ is merely hypothetical. If, however, we succeed in calculating the mean value of the difference between the m_{kl} actually found and the hypothetical true value $\overline{m}_{kl}$, we are enabled to estimate the uncertainty of the m_{kl} actually found.

A certain set of observations (for instance the actual one) giving for m_{kl} the deviation

$$\Delta m_{kl} = m_{kl} - \overline{m}_{kl}$$

from the true value (which deviation may be called the error of m_{kl}), the square of the mean error of m_{kl} is:

$$\mathrm{E}^2(m_{kl}) = \overline{\Delta m_{kl}{}^2}.$$

The set of observations being repeated hypothetically, a function $\varphi(m_{kl})$ of the magnitudes m_{kl} will oscillate about a mean value $\overline{\varphi}$ and will have — in each particular set of observations — an error $\Delta\varphi = \varphi - \overline{\varphi}$ depending from the errors Δm_{kl} according to the formula

$$\Delta\varphi = \sum_{k=1}^{n}\sum_{l=1}^{n}\frac{\partial\varphi}{\partial m_{kl}}\cdot\Delta m_{kl},$$

the coefficients $\frac{\partial\varphi}{\partial m_{kl}}$ of this formula having properly to be calculated by means of the true values of m_{kl}.

The square of the mean error of φ is

$$\mathrm{E}^2(\varphi) = \overline{\Delta\varphi^2} = \overline{\left(\sum_{k=1}^{n}\sum_{l=1}^{n}\frac{\partial\varphi}{\partial m_{kl}}\Delta m_{kl}\right)^2} = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n}\frac{\partial\varphi}{\partial m_{ij}}\,\frac{\partial\varphi}{\partial m_{kl}}\,\overline{\Delta m_{ij}\,\Delta m_{kl}}.$$

The values of m_{ij}, m_{kl} to be substituted in the coefficients of $\overline{\Delta m_{ij}\,\Delta m_{kl}}$ must also be the true ones.

Now it is necessary in the first place to determine the magnitude $\overline{\Delta m_{ij}\,\Delta m_{kl}}$.

$$\Delta m_{ij} = m_{ij} - \overline{m}_{ij} \quad , \quad \Delta m_{kl} = m_{kl} - \overline{m}_{kl},$$

thus

$$\overline{\Delta m_{ij}\,\Delta m_{kl}} = \overline{(m_{ij} - \overline{m}_{ij})(m_{kl} - \overline{m}_{kl})} = \overline{m_{ij}\,m_{kl}} - \overline{m}_{ij}\,.\,\overline{m}_{kl}.$$

We have further

$$\overline{m_{ij}\,m_{kl}} = \overline{\frac{[u_i\,u_j]}{N}\cdot\frac{[u_k\,u_l]}{N}} = \overline{\frac{[u_i\,u_j\,u_k\,u_l] + [[u_i\,u_j\,u_k'\,u_l']]}{N^2}}.$$

In the sum [ ] each of the N terms corresponds to one distinct individual. The sum [[ ]] on the contrary, runs over the $N(N-1)$ terms $u_i\,u_j\,u_k'\,u_l'$, the factors $u_k'\,u_l'$ of which correspond to another individual than the factors $u_i\,u_j$. As in repeating the set of observations, each individual undergoes changes to be considered as independent of the changes of another individual, we may consider the two products $u_i\,u_j$ and $u_k'\,u_l'$ in the sum $[[u_i\,u_j\,u_k'\,u_l']]$ as independent of each other, and we may therefore write

$$\overline{[[u_i\,u_j\,u_k'\,u_l']]} = N(N-1)\,\overline{m}_{ij}\,.\,\overline{m}_{kl}\,.$$

Hence we obtain

$$\overline{m_{ij}\; m_{kl}} = \frac{\overline{\frac{1}{N}\,[u_i\; u_j\; u_k\; u_l]}}{N} + \left(1 - \frac{1}{N}\right) \overline{m}_{ij} \,.\, \overline{m}_{kl}\;;$$

and

$$\overline{\Delta\, m_{ij}\; \Delta\, m_{kl}} = \frac{\overline{\frac{1}{N}\,[u_i\; u_j\; u_k\; u_l]} - \overline{m}_{ij} \,.\, \overline{m}_{kl}}{N}\,,$$

or, since $\frac{1}{N}[u_i\; u_j\; u_k\; u_l]$ is the empirical value of m_{ijkl},

$$\overline{\Delta\, m_{ij}\; \Delta\, m_{kl}} = \frac{\overline{m}_{ijkl} - \overline{m}_{ij} \,.\, \overline{m}_{kl}}{N} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

We can put (7), by means of (6), in the form

$$\overline{\Delta\, m_{ij}\; \Delta\, m_{kl}} = \frac{1}{N}\,(\overline{m}_{ik} \,.\, \overline{m}_{jl} + \overline{m}_{il} \,.\, \overline{m}_{jk})\,,$$

or, replacing the essentially unknown "true" values $\overline{m}_{ik}$, etc., by the values m_{ik}, etc., actually found (the only ones we have at our disposal) which at any rate approximate to the true values,

$$\overline{\Delta\, m_{ij}\; \Delta\, m_{kl}} = \frac{1}{N}\,(m_{ik}\; m_{jl} + m_{il}\; m_{jk}) \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{II}^a)$$

This fundamental formula will prove useful in calculating all the mean errors desired.

We shall, firstly, as an application of (II^a), derive the mean error of m_{kk}. For this we find

$$\mathrm{E}^2\,(m_{kk}) = \overline{\Delta\, m_{kk}{}^2} = \frac{2}{N}\, m_{kk}{}^2,$$

thus

$$\mathrm{E}\;(m_{kk}) = m_{kk} \,.\, \sqrt{\frac{2}{N}}\,.$$

m_{kk} being the square of the standard deviation ε_k of u_k, so that

$$\varepsilon_k = \sqrt{m_{kk}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8^A)$$

we have

$$\Delta\, \varepsilon_k = \frac{\Delta\, m_{kk}}{2\sqrt{m_{kk}}}\,,$$

hence also

$$\mathrm{E}\;(\varepsilon_k) = \frac{\mathrm{E}\;(m_{kk})}{2\sqrt{m_{kk}}} = \sqrt{\frac{m_{kk}}{2N}}\,.$$

from which derives the well-known formula

$$\mathrm{E}\,(\varepsilon_k) = \frac{\varepsilon_k}{\sqrt{2N}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (8^B)$$

We shall, secondly, calculate the mean error of the coefficient of correlation of second order:

$$\gamma_{kl} = \frac{m_{kl}}{\sqrt{m_{kk}\, m_{ll}}} . \quad . \; . \; . \; . \; . \; . \; . \; . \quad (9^A)$$

$$\Delta\,(\gamma_{kl}^2) = \frac{m_{kk}\, m_{ll} \,.\, 2\, m_{kl}\, \Delta\, m_{kl} - m_{kl}^2 .\, m_{ll}\, \Delta\, m_{kk} - m_{kl}^2 .\, m_{kk} \Delta\, m_{ll}}{m_{kk}^2\, m_{ll}^2},$$

$$\Delta \gamma_{kl} = \frac{\Delta\,(\gamma_{kl}^2)}{2\,\gamma_{kl}} = \frac{-\, m_{kl}\, m_{ll}\, \Delta\, m_{kk} + 2\, m_{kk}\, m_{ll}\, \Delta\, m_{kl} - m_{kk}\, m^{kl}\, \Delta\, m_{l}}{2\,(m_{kk}\, m_{ll})^{3/2}}$$

$$\mathrm{E}^2\,(\gamma_{kl}) = \overline{\Delta\,\gamma_{kl}^{\;2}} = \frac{1}{4\; m_{kk}^3\; m_{ll}^3}\, [m_{kl}^2\, m_{ll}^2\, \overline{\Delta\, m_{kk}^2} + m_{kk}^2\, m_{kl}^2\, \overline{\Delta\, m_{ll}^2} +$$

$$+ 2\, m_{kk}\, m_{kl}^2\, m_{ll}\, \overline{\Delta\, m_{kk}\, \Delta\, m_{ll}} + 4\, m_{kk}^2\, m_{ll}^2\, \overline{\Delta\, m_{kl}^2} -$$

$$- 4\, m_{kk}\, m_{kl}\, m_{ll}\, (m_{ll}\, \overline{\Delta\, m_{kl} \Delta\, m_{kk}} + m_{kk}\, \overline{\Delta\, m_{kl}\, \Delta\, m_{ll}}\,)\,]$$

or, by means of (II^a),

$$\mathrm{E}^2\,(\gamma_{kl}) = \frac{1}{4\;N m_{kk}^3\, m_{ll}^3}\, [m_{kl}^2\, m_{ll}^2 \,.\, 2\, m_{kk}^2 + m_{kk}^2 m_{kl}^2 \,.\, 2\, m_{ll}^2 +$$

$$+ 2\, m_{kk}\, m_{kl}^2\, m_{ll} \,.\, 2\, m_{kl}^2 + 4\, m_{kk}^2\, m_{ll}^2\, (m_{kk}\, m_{ll} + m_{kl}^2) -$$

$$- 4\, m_{kk}\, m_{kl}\; m_{ll}\, (m_{ll} \,.\, 2\, m_{kl}\; m_{kk} + m_{kk} \,.\, 2\, m_{kl}\; m_{ll}\,)\,]$$

$$= \frac{(m_{kk}\, m_{ll} - m_{kl}^2)^2}{N\, m_{kk}^2\, m_{ll}^2} = \frac{1}{N}\,(1 - \gamma_{kl}^2)^2,$$

whence the equally well-known formula

$$\mathrm{E}\,(\gamma_{kl}) = \frac{1 - \gamma_{kl}^2}{\sqrt{N}} = \frac{1}{\sqrt{N}} . \frac{m_{kk}\, m_{ll} - m_{kl}^2}{m_{kk}\, m_{ll}} \quad . \; . \; . \; . \quad (9^B)$$

is found.

From $\sum\limits_{\alpha=1}^{n} p_{i\alpha}\, m_{\alpha\beta} = \omega_{i\beta}$ (see (5))

ensues

$$\sum_{\alpha=1}^{n} m_{\alpha\beta}\, \Delta\, p_{i\alpha} + \sum_{\alpha=1}^{n} p_{i\alpha}\, \Delta\, m_{\alpha\beta} = 0,$$

or

$$\sum_{\alpha=1}^{n} m_{\alpha\beta}\, \Delta\, p_{i\alpha} = - \sum_{\alpha=1}^{n} p_{i\alpha}\, \Delta\, m_{\alpha\beta} .$$

Further

$$\sum_{\beta=1}^{n} p_{j\beta}\, (\sum_{\alpha=1}^{n} m_{\alpha\beta}\, \Delta\, p_{i\alpha}) = - \sum_{\alpha=1}^{n}\; \sum_{\beta=1}^{n} p_{i\alpha}\; p_{j\beta}\, \Delta\, m_{\alpha\beta},$$

or

$$\sum_{\alpha=1}^{n} \left(\sum_{\beta=1}^{n} p_{j\beta}\, m_{\alpha\beta}\right) \triangle p_{i\alpha} = -\sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} p_{i\alpha}\, p_{j\beta} \triangle m_{\alpha\beta},$$

or

$$\sum_{\alpha=1}^{n} \omega_{j\alpha} \triangle p_{i\alpha} = -\sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} p_{i\alpha}\, p_{j\beta} \triangle m_{\alpha\beta},$$

or

$$\triangle p_{ij} = -\sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} p_{i\alpha}\, p_{j\beta} \triangle m_{\alpha\beta}.$$

Likewise

$$\triangle p_{kl} = -\sum_{\gamma=1}^{n} \sum_{\delta=1}^{n} p_{k\gamma}\, p_{l\delta} \triangle m_{\gamma\delta},$$

whence

$$\overline{\triangle p_{ij} \triangle p_{kl}} = \sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} \sum_{\gamma=1}^{n} \sum_{\delta=1}^{n} p_{i\alpha}\, p_{j\beta}\, p_{k\gamma}\, p_{l\delta} \,.\, \overline{\triangle m_{\alpha\beta} \triangle m_{\gamma\delta}}$$

$$= \frac{1}{N} \sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} \sum_{\gamma=1}^{n} \sum_{\delta=1}^{n} p_{i\alpha}\, p_{j\beta}\, p_{k\gamma}\, p_{l\delta} \left(m_{\alpha\gamma}\, m_{\beta\delta} + m_{\alpha\delta}\, m_{\beta\gamma}\right)$$

$$= \frac{1}{N} \sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} p_{i\alpha}\, p_{j\beta} \left(\sum_{\gamma=1}^{n} p_{k\gamma}\, m_{\alpha\gamma} \,.\, \sum_{\delta=1}^{n} p_{l\delta}\, m_{\beta\delta} + \sum_{\gamma=1}^{n} p_{k\gamma}\, m_{\beta\gamma} \,.\, \sum_{\delta=1}^{n} p_{l\delta}\, m_{\alpha\delta}\right)$$

$$= \frac{1}{N} \sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} p_{i\alpha}\, p_{j\beta} \left(\omega_{k\alpha}\, \omega_{l\beta} + \omega_{k\beta}\, \omega_{l\alpha}\right)$$

or

$$\overline{\triangle p_{ij} \triangle p_{kl}} = \frac{1}{N} \left(p_{ik}\, p_{jl} + p_{il}\, p_{jk}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (\mathrm{II}^b)$$

So the magnitudes p_{kl} appear to satisfy the same law of error as the magnitudes m_{kl}.

From

$$\triangle p_{kl} = -\sum_{\gamma=1}^{n} \sum_{\delta=1}^{n} p_{k\gamma}\, p_{l\delta} \triangle m_{\gamma\delta}$$

we have yet

$$\overline{\triangle p_{kl} \triangle m_{ij}} = -\sum_{\gamma=1}^{n} \sum_{\delta=1}^{n} p_{k\gamma}\, p_{l\delta}\, \overline{\triangle m_{\gamma\delta}}\, \overline{\triangle m_{ij}} =$$

$$= -\frac{1}{N} \sum_{\gamma=1}^{n} \sum_{\delta=1}^{n} p_{k\gamma}\, p_{l\delta} \left(m_{i\gamma}\, m_{j\delta} + m_{i\delta}\, m_{j\gamma}\right) = -\frac{1}{N} \left(\omega_{ik}\, \omega_{jl} + \omega_{il}\, \omega_{jk}\right);$$

and likewise

$$\overline{\triangle p_{ij} \triangle m_{kl}} = -\frac{1}{N} \left(\omega_{ik}\, \omega_{jl} + \omega_{il}\, \omega_{jk}\right);$$

whence

$$\overline{\triangle p_{ij} \triangle m_{kl}} = \overline{\triangle p_{kl} \triangle m_{ij}} = -\frac{1}{N} \left(\omega_{ik}\, \omega_{jl} + \omega_{il}\, \omega_{jk}\right) \quad . \quad . \quad (\mathrm{II}^c)$$

$$\Delta M = \sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} M_{\alpha\beta} \, \Delta m_{\alpha\beta}$$

$$\overline{\Delta M \, \Delta m_{kl}} = \sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} M_{\alpha\beta} \, \overline{\Delta m_{\alpha\beta} \, \Delta m_{kl}} = \frac{M}{N} \sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} p_{\alpha\beta} (m_{k\alpha} m_{l\beta} + m_{k\beta} m_{l\alpha})$$

$$= \frac{M}{N} \sum_{\alpha=1}^{n} \left(m_{k\alpha} \sum_{\beta=1}^{n} p_{\alpha\beta} \, m_{l\beta} + m_{l\alpha} \sum_{\beta=1}^{n} p_{\alpha\beta} \, m_{k\beta} \right)$$

$$= \frac{M}{N} \sum_{\alpha=1}^{n} (m_{k\alpha} \, \omega_{l\alpha} + m_{l\alpha} \, \omega_{k\alpha}),$$

or

$$\overline{\Delta M \, \Delta m_{kl}} = \frac{2\,M}{N} \, m_{kl} \quad \ldots\ldots\ldots \quad (10^{a})$$

Likewise

$$\overline{\Delta P \, . \, \Delta p_{kl}} = \frac{2\,P}{N} \, p_{kl} \quad \ldots\ldots\ldots \quad (10^{b})$$

$$\overline{\Delta M^2} = \sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} M_{\alpha\beta} \, \overline{\Delta M \, \Delta m_{\alpha\beta}} = \sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} M \, p_{\alpha\beta} \, . \, \frac{2\,M}{N} \, m_{\alpha\beta} =$$

$$= \frac{2\,M^2}{N} \sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} p_{\alpha\beta} \, m_{\alpha\beta},$$

or, since

$$\sum_{\alpha=1}^{n} \sum_{\beta=1}^{n} p_{\alpha\beta} \, m_{\alpha\beta} = \sum_{\alpha=1}^{n} \omega_{\alpha\alpha} = n,$$

$$\overline{\Delta M^2} = \frac{2\,n\,M^2}{N} \quad \ldots\ldots\ldots\ldots \quad (11^{a})$$

Likewise

$$\overline{\Delta P^2} = \frac{2\,n\,P^2}{N} \quad \ldots\ldots\ldots\ldots \quad (11^{b})$$

$$\overline{\Delta M \, \Delta M_{kl}} = \overline{\Delta M \, (p_{kl} \, \Delta M + M \, \Delta p_{kl})} = p_{kl} \, \overline{\Delta M^2} + M \, . \, \overline{\Delta M \, \Delta p_{kl}} .$$

Now

$$\Delta M = -\frac{1}{P^2} \, \Delta P = -M^2 \, \Delta P,$$

thus

$$\overline{\Delta M \, \Delta p_{kl}} = -M^2 \, \overline{\Delta P \, . \, \Delta p_{kl}} = -M^2 \, . \, \frac{2P}{N} \, p_{kl} = -\frac{2M}{N} \, p_{kl},$$

whence

$$\overline{\Delta M \, \Delta M_{kl}} = p_{kl} \, \frac{2n\,M^2}{N} - \frac{2\,M^2}{N} \, p_{kl} = \frac{2\,(n-1)\,M^2}{N} \, p_{kl},$$

or

$$\overline{\Delta M \, \Delta M_{kl}} = \frac{2(n-1)\,M\,M_{kl}}{N} \quad \ldots\ldots\ldots \quad (12^{a})$$

Likewise

$$\overline{\Delta P \,.\, \Delta P_{kl}} = \frac{2\,(n-1)\,P\,P_{kl}}{N} \quad . \; . \; . \; . \; . \; . \; . \quad (12^b)$$

$$\overline{\Delta M_{ij}\;\Delta m_{kl}} = \overline{(p_{ij}\,\Delta M + M\,\Delta p_{ij})\;\Delta m_{kl}} = p_{ij}\;\overline{\Delta M\;\Delta m_{kl}} + M\;\overline{\Delta p_{ij}\;\Delta m_{kl}}$$

$$= p_{ij}\,\frac{2M}{N}\,m_{kl} + M\,.\,-\frac{\omega_{ik}\,\omega_{jl} + \omega_{il}\,\omega_{jk}}{N}$$

or

$$\overline{\Delta M_{ij}\;\Delta m_{kl}} = \frac{1}{N}\left\{2\,M_{ij}\,m_{kl} - M\,(\omega_{ik}\,\omega_{jl} + \omega_{il}\,\omega_{jk})\right\} \quad . \; . \quad (13^a)$$

in particular

$$\overline{\Delta M_{kl}\,\Delta m_{kl}} = \frac{1}{N}\,(2\,M_{kl}\,m_{kl} - M) \qquad \text{voor} \qquad l \neq k,$$

$$\overline{\Delta M_{kk}\,\Delta m_{kk}} = \frac{2}{N}\,(M_{kk}\,m_{kk} - M).$$

Likewise

$$\overline{\Delta P_{ij}\;\Delta p_{kl}} = \frac{1}{N}\left\{2\,P_{ij}\,p_{kl} - P\,(\omega_{ik}\,\omega_{jl} + \omega_{il}\,\omega_{jk})\right\} \quad . \; . \; . \quad (13^b)$$

$$\overline{\Delta M_{ij}\,\Delta M_{kl}} = \overline{(p_{ij}\,\Delta M + M\,\Delta p_{ij})\,(p_{kl}\,\Delta M + M\,\Delta p_{kl})}$$

$$= p_{ij}\,p_{kl}\,\overline{\Delta M^2} + M\,(p_{kl}\,\overline{\Delta M\;\Delta p_{ij}} + p_{ij}\;\overline{\Delta M\;\Delta p_{kl}}) + M^2\,\overline{\Delta p_{ij}\,\Delta p_{kl}}$$

$$= p_{ij}\,p_{kl}\,\frac{2n\,M^2}{N} + M\left(p_{kl}\,.\,-\frac{2M}{N}\,p_{ij} + p_{ij}\,.\,-\frac{2M}{N}\,p_{kl}\right) + M^2\,\frac{p_{ik}\,p_{jl} + p_{il}\,p_{jk}}{N}$$

$$= \frac{2n}{N}\,M_{ij}\,M_{kl} - \frac{4}{N}\,M_{ij}\,M_{kl} + \frac{M_{ik}\,M_{jl} + M_{il}\,M_{jk}}{N},$$

or

$$\overline{\Delta M_{ij}\,\Delta M_{kl}} = \frac{1}{N}\left\{2\,(n-2)\,M_{ij}\,M_{kl} + M_{ik}\,M_{jl} + M_{il}\,M_{jk}\right\} \quad . \quad (14^a)$$

Likewise

$$\overline{\Delta P_{ij}\;\Delta P_{kl}} = \frac{1}{N}\left\{2\,(n-2)\,P_{ij}\,P_{kl} + P_{ik}\,P_{jl} + P_{il}\,P_{jk}\right\} \quad . \quad (14^b)$$

As a further application we shall calculate:
1⁰ the mean error of p_{kk}.

$$\mathsf{E}^2\,(p_{kk}) = \overline{\Delta p_{kk}{}^2} = \frac{2}{N}\,p_{kk}{}^2,$$

thus

$$\mathsf{E}\,(p_{kk}) = \sqrt{\frac{2}{N}}\cdot p_{kk} = \sqrt{\frac{2}{N}}\cdot\frac{M_{kk}}{M}.$$

Putting $p_{kk} = 2\,h_k{}^2$

or

$$h_k = \sqrt{\frac{p_{kk}}{2}} = \sqrt{\frac{M_{kk}}{2M}} \quad . \; . \; . \; . \; . \; . \; . \quad (15^A)$$

we find

$$\mathrm{E}\,(2\,h_k{}^2) = 4\,h_k\,\mathrm{E}\,(h_k) = \sqrt{\frac{2}{N}}\,.\,2\,h_k{}^2,$$

or

$$\mathrm{E}\,(h_k) = \frac{1}{\sqrt{2N}}\,h_k = \frac{1}{2\sqrt{N}}\,.\sqrt{\frac{M_{kk}}{M}}\;.\;.\;.\;.\;.\;.\quad (15^B)$$

2^0 the mean error of the magnitude λ_{kl}, defined by

$$\lambda_{kl} = \frac{p_{kl}}{\sqrt{p_{kk}\,p_{ll}}}\;,\quad (\lambda_{kk} = 1)\;.\;.\;.\;.\;.\;.\;.\quad (16^A)$$

which is built up of the magnitudes p_{kl} in the same manner as γ_{kl} of the magnitudes m_{kl}.

Since the magnitudes p_{kl} follow the same law of error as the m_{kl}, we find, analogically to (9^B):

$$\mathrm{E}\,(\lambda_{kl}) = \frac{1-\lambda_{kl}{}^2}{\sqrt{N}}\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\quad (16^B)$$

Here we have

$$1-\lambda_{kl}{}^2 = \frac{p_{kk}\,p_{ll} - p_{kl}{}^2}{p_{kk}\,p_{ll}} = \frac{M_{kk}\,M_{ll} - M_{kl}{}^2}{M_{kk}\,M_{ll}} = \frac{M\,.\,M_{kk,ll}}{M_{kk}\,M_{ll}}\;,$$

thus

$$\mathrm{E}\,(\lambda_{kl}) = \frac{1}{\sqrt{N}}\,.\,\frac{M\,.\,M_{kk,ll}}{M_{kk}\,M_{ll}}\;.\;.\;.\;.\;.\;.\quad (16^B)\;bis$$

Analogically to

$$m_{kl} = \gamma_{kl}\,\varepsilon_k\,\varepsilon_l \qquad (\gamma_{kk} = 1)\;.\;.\;.\;.\;.\;.\;.\quad (17^a)$$

we may write for the coefficient p_{kl}:

$$p_{kl} = 2\,\lambda_{kl}\,h_k\,h_l \qquad (\lambda_{kk} = 1)\quad .\;.\;.\;.\;.\;.\quad (17^b)$$

$-\lambda_{kl}$ is also called the partial coefficient of correlation between u_k and u_l of order $n-2$, i.e. the coefficient of correlation between u_k and u_l, the remaining $n-2$ variables $u_1 \ldots . (u_k\,u_l) \ldots . u_n$ being kept constant. Sometimes it is designated by the symbol of the coefficient of correlation, the subscripts of the variables kept constant being placed after the subscripts [1]), k, l, thus

$$-\lambda_{kl} = \gamma_{kl:1\ldots(kl)\ldots n}\,,$$

or, since the former subscripts (primary subscripts) as a matter of course do not appear among the subscripts on the right (secondary subscripts),

$$-\lambda_{kl} = \gamma_{kl:1\ldots n}.$$

In the ordinary coefficient of correlation all the $n-2$ remaining variables are left out of account, that is to say: their variability is taken into account to its full extent. The analytical meaning of this is integration along these variables.

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[1]) See G. UDNY YULE: An Introduction to the Theory of Statistics, London, Griffin & Co., p. 229 ff.

By integrating only along one variable, e.g. u_α, we retain a formula of distribution in the $n-1$ variables u_k, $k \neq \alpha$. Denoting the coefficients of the homogeneous quadratic form (which before integrating along u_α, were $\frac{1}{2}\, p_{kl}$) in this case by

$$\tfrac{1}{2}\, p_{kl}^{(\alpha)} = \lambda_{kl}^{(\alpha)} \,.\, h_k^{(\alpha)} .\, h_l^{(\alpha)} \qquad (k \neq \alpha, \quad l \neq \alpha),$$

whence

$$h_k^{(\alpha)} = \sqrt{\frac{p_{kk}^{(\alpha)}}{2}} \quad , \quad \lambda_{kl}^{(\alpha)} = \frac{p_{kl}^{(\alpha)}}{\sqrt{p_{kk}^{(\alpha)} \,.\, p_{ll}^{(\alpha)}}} \qquad (\lambda_{kk}^{(\alpha)} = 1),$$

then $-\lambda_{kl}^{(\alpha)}$ is a partial coefficient of correlation between u_k en u_l of order $n-3$, which we may also designate by

$$-\lambda_{kl}^{(\alpha)} = \gamma_{kl;\, 1, \dots (\alpha) \dots n}.$$

We find

$$p_{kl}^{(\alpha)} = p_{kl} - \frac{p_{\alpha k}\, p_{\alpha l}}{p_{\alpha\alpha}} = \frac{M_{\alpha\alpha}\, M_{kl} - M_{\alpha k}\, M_{\alpha l}}{M\, M_{\alpha\alpha}} = \frac{M_{\alpha\alpha,\, kl}}{M_{\alpha\alpha}}$$

and therefore

$$\lambda_{kl}^{(\alpha)} = \frac{\lambda_{kl} - \lambda_{\alpha k}\, \lambda_{\alpha l}}{\sqrt{1 - \lambda_{\alpha k}{}^2} \,.\, \sqrt{1 - \lambda_{\alpha l}{}^2}}.$$

Because the magnitudes $p_{kl}^{(\alpha)}$ in the formula of distribution for the $n-1$ variables u_k $(k \neq \alpha)$ play the same part as the p_{kl} in the complete formula of distribution, we have also for the $p_{kl}^{(\alpha)}$:

$$\overline{\triangle p_{ij}^{(\alpha)} \,.\, \triangle p_{kl}^{(\alpha)}} = \frac{1}{N} \left(p_{ki}^{(\alpha)} \,.\, p_{jl}^{(\alpha)} + p_{il}^{(\alpha)} .\, p_{jk}^{(\alpha)}\right).$$

The mean error of $\lambda_{kl}^{(\alpha)}$ too is, in the same way, connected with $\lambda_{kl}^{(\alpha)}$ as $\mathrm{E}\,(\lambda_{kl})$ with λ_{kl}; hence

$$\mathrm{E}\;(\lambda_{kl}^{(\alpha)}) = \frac{1 - \lambda_{kl}^{(\alpha)\,2}}{\sqrt{N}}.$$

Since

$$\lambda_{kl}^{(\alpha)\,2} = \frac{p_{kl}^{(\alpha)\,2}}{p_{kk}^{(\alpha)} \,.\, p_{ll}^{(\alpha)}} = \frac{\left(\dfrac{M_{\alpha\alpha,\, kl}}{M_{\alpha\alpha}}\right)^2}{\dfrac{M_{\alpha\alpha,\, kk}}{M_{\alpha\alpha}} \,.\, \dfrac{M_{\alpha\alpha,\, ll}}{M_{\alpha\alpha}}} = \frac{M_{\alpha\alpha,\, kl}{}^2}{M_{\alpha\alpha,\, kk} \,.\, M_{\alpha\alpha,\, ll}},$$

and

$$1 - \lambda_{kl}^{(\alpha)\,2} = \frac{M_{\alpha\alpha,\, kk} \,.\, M_{\alpha\alpha,\, ll} - M_{\alpha\alpha,\, kl}{}^2}{M_{\alpha\alpha,\, kk} \,.\, M_{\alpha\alpha,\, ll}} = \frac{M_{\alpha\alpha} \,.\, M_{\alpha\alpha,\, kk,\, ll}}{M_{\alpha\alpha, kk} \,.\, M_{\alpha\alpha,\, ll}},$$

we have

$$\gamma_{kl:\, 1, \dots (\alpha) \dots n} = -\lambda_{kl}^{(\alpha)} = \frac{-M_{\alpha\alpha,\, kl}}{\sqrt{M_{\alpha\alpha,\, kk} \,.\, M_{\alpha\alpha,\, ll}}} \quad . \quad . \quad . \quad (16^A)'$$

with

$$\mathrm{E}\;(\lambda_{kl}^{(\alpha)}) = \frac{1 - \lambda_{kl}^{(\alpha)\,2}}{\sqrt{N}} = \frac{1}{\sqrt{N}} \,.\, \frac{M_{\alpha\alpha} \,.\, M_{\alpha\alpha,\, kk,\, ll}}{M_{\alpha\alpha,\, kk} \,.\, M_{\alpha\alpha,\, ll}} \quad . \quad . \quad . \quad . \quad . \quad (16^B)'$$

We have also to deal with the partial standard deviation of order $n-1$ (bedingte Streuung), i. e. the standard deviation of the variable u_k, the $u_l (l \neq k)$ being kept constant.

Representing this by $\varepsilon_{k;1\ldots(k)\ldots n}$, or, more simply, by $\varepsilon_{k;1\ldots n}$, we have

$$\varepsilon^2_{k;1\ldots n} = \frac{1}{2h_k^2} = \frac{1}{p_{kk}} = \frac{M}{M_{kk}},$$

or

$$\varepsilon_{k;1\ldots n} = \frac{1}{h_k\sqrt{2}} = \sqrt{\frac{M}{M_{kk}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (15^A)\ \mathit{bis}$$

So the mean error of $\varepsilon_{k;1\ldots n}$ is, according to (15^B),

$$\mathrm{E}\,(\varepsilon_{k;1\ldots n}) = \frac{1}{h_k^2\sqrt{2}} \, . \, \mathrm{E}\,(h_k) = \frac{1}{h_k^2\sqrt{2}} \cdot \frac{h_k}{\sqrt{2N}},$$

or

$$\mathrm{E}\,(\varepsilon_{k;1\ldots n}) = \frac{\varepsilon_{k;1\ldots n}}{\sqrt{2N}} = \sqrt{\frac{M}{2\,N\,M_{kk}}} \quad . \quad . \quad . \quad . \quad . \quad (15^B)\ \mathit{bis}$$

A partial standard deviation of order $n-2$ (for instance of u_k) is obtained by keeping constant only $n-2$ variables, and leaving only one, for instance $u_\alpha\ (\alpha \neq k)$, out of account, or, analytically spoken: by integrating along u_α.

Then we find

$$\varepsilon_{k;1\ldots(\alpha)\ldots n} = \frac{1}{\sqrt{p_{kk}^{(\alpha)}}} = \frac{1}{h_k^{(\alpha)} . \sqrt{2}} = \sqrt{\frac{M_{\alpha\alpha}}{M_{\alpha\alpha,\,kk}}}, \quad . \quad . \quad (15^A)'$$

with

$$\mathrm{E}\,(\varepsilon_{k;1\ldots(\alpha)\ldots n}) = \frac{\varepsilon_{k;1\ldots(\alpha)\ldots n}}{\sqrt{2\,N}} = \sqrt{\frac{M_{\alpha\alpha}}{2\,N\,M_{\alpha\alpha,\,kk}}} \quad . \quad . \quad . \quad (15^B)'$$

By carrying on this process, i.e. by integrating, besides along u_α, also along $u_\beta\ (\beta \neq k)$, we obtain the partial standard deviation of order $n-3$:

$$\varepsilon_{k;1\ldots(\alpha\beta)\ldots n} = \frac{1}{\sqrt{p_{kk}^{(\alpha\beta)}}} = \frac{1}{h_k^{(\alpha\beta)} . \sqrt{2}} = \sqrt{\frac{M_{\alpha\alpha,\,\beta\beta}}{M_{\alpha\alpha,\,\beta\beta,\,kk}}}, \quad . \quad (15^A)''$$

with

$$\mathrm{E}\,(\varepsilon_{k;1\ldots(\alpha\beta)\ldots n}) = \frac{\varepsilon_{k;1\ldots(\alpha\beta)\ldots n}}{\sqrt{2\,N}} = \sqrt{\frac{M_{\alpha\alpha,\,\beta\beta}}{2\,N\,M_{\alpha\alpha,\,\beta\beta,\,kk}}} \quad . \quad (15^B)''$$

and the partial coefficient of correlation of order $n-4$:

$$\gamma_{kl;1\ldots(\alpha\beta)\ldots n} = -\,\lambda_{kl}^{(\alpha\beta)} = \frac{+\,M_{\alpha\alpha,\,\beta\beta,\,kl}}{\sqrt{M_{\alpha\alpha,\,\beta\beta,\,kk} \, . \, M_{\alpha\alpha,\,\beta\beta,\,ll}}} \quad . \quad (16^A)''$$

with

$$\mathrm{E}\,(\gamma_{kl;1\ldots(\alpha\beta)\ldots n}) = \frac{1}{\sqrt{N}} \cdot \frac{M_{\alpha\alpha,\,\beta\beta} \, . \, M_{\alpha\alpha,\,\beta\beta,\,kk,\,ll}}{M_{\alpha\alpha,\,\beta\beta,\,kk} \, . \, M_{\alpha\alpha,\,\beta\beta,\,ll}} \quad . \quad . \quad . \quad (16^B)''$$

Integrating along $u_\alpha, u_\beta, u_\gamma, \ldots$ (all different from k and l) we only have, starting from the formula for the complete set of variables, to free the determinants M, M_{kk}, M_{kl}, M_{ll}, $M_{kk,ll}$ of the rows and columns corresponding to the subscribts $\alpha, \beta, \gamma, \ldots$

Proceeding in this way to the end, then in $(15^A)''$ and $(15^B)''$:

$M_{\alpha\alpha,\beta\beta,\gamma\gamma\ldots}$ ends in m_{kk}, $M_{\alpha\alpha,\beta\beta,\gamma\gamma,\ldots,kk}$ in 1, and in $(16^A)''$ and $(16^B)''$:
$M_{\alpha\alpha,\beta\beta,\gamma\gamma,\ldots}$ ends in $m_{kk}\, m_{ll} - m_{kl}{}^2$, $M_{\alpha\alpha,\beta\beta,\gamma\gamma,\ldots kk}$ in m_{ll}, $M_{\alpha\alpha,\beta\beta,\gamma\gamma,\ldots kl}$ in $-m_{kl}$, $M_{\alpha\alpha,\beta\beta,\gamma\gamma,\ldots ll}$ in m_{kk} and $M_{\alpha\alpha,\beta\beta,\gamma\gamma,\ldots kk,ll}$ in 1.

So we find for the standard deviation of order 1

$$\varepsilon_k = \sqrt{\frac{m_{kk}}{1}} \quad \text{with} \quad \mathrm{E}\,(\varepsilon_k) = \frac{\varepsilon_k}{\sqrt{2N}},$$

and for the coefficient of correlation of order 2:

$$\gamma_{kl} = \frac{+\,m_{kl}}{\sqrt{m_{ll}\, m_{kk}}} \quad \text{with} \quad \mathrm{E}\,(\gamma_{kl}) = \frac{1}{\sqrt{N}} \cdot \frac{m_{kk}\, m_{ll} - m_{kl}{}^2}{m_{ll}\, m_{kk}} = \frac{1}{\sqrt{N}}(1 - \gamma_{kl}{}^2)$$

in conformity with the formulae $(8^{A,B}, 9^{A,B})$.

Finally we will treat of the coefficients of regression.

Resolving the equation

$$\frac{\partial f}{\partial u_i} = \sum_{k=1}^{n} p_{ik}\, u_k = 0$$

with respect to u_i, we obtain the so-called equation of regression for u_i, viz.:

$$u_i = -\sum_{\substack{k=1\\k \neq i}}^{n} \frac{p_{ik}}{p_{ii}} \cdot u_k = \sum_{k \neq i} \varrho_{i\,|\,k;\,1\ldots n} \cdot u_k\,.$$

The coefficient

$$\varrho_{i\,|\,k;\,1\ldots n} = -\frac{p_{ik}}{p_{ii}} = -\frac{\lambda_{ik}\, h_k}{h_i} = -\frac{M_{ik}}{M_{ii}} = \gamma_{ik;\,1\ldots n} \cdot \frac{\varepsilon_{i;\,1\ldots n}}{\varepsilon_{k;\,1\ldots n}}\,, \qquad (18^A)$$

is called the coefficient of regression of order $n-2$ of u_i with respect to u_k.

Obviously the subscripts i and k may not be changed.

From

$$\Delta\, \varrho_{i\,|\,k;\,1\ldots n} = \frac{p_{ik}\, \Delta\, p_{ii} - p_{ii}\, \Delta\, p_{ik}}{p_{ii}^2}$$

ensues

$$\mathrm{E}^2\,(\varrho_{i\,|\,k;\,1\ldots n}) = \overline{(\Delta\varrho_{i\,|\,k;\,1\ldots n})^2} = \frac{p_{ik}{}^2 \cdot \overline{\Delta\, p_{ii}}{}^2 - 2\, p_{ii}\, p_{ik} \cdot \overline{\Delta p_{ii}\, \Delta\, p_{ik}} + p_{ii}{}^2 \cdot \overline{\Delta\, p_{ik2}}}{p_{ii}{}^4}$$

$$= \frac{p_{ik}{}^2 \cdot 2 p_{ii}{}^2 - 2\, p_{ii} p_{ik} \cdot 2 p_{ii} p_{ik} + p_{ii}{}^2\,(p_{ii} p_{kk} + p_{ik}{}^2)}{N p_{ii}{}^4}$$

$$= \frac{p_{ii}\, p_{kk} - p_{ik}{}^2}{N\, p_{ii}{}^2} = \frac{M_{ii}\, M_{kk} - M_{ik}{}^2}{N\, M_{ii}{}^2} = \frac{M \,.\, M_{ii,kk}}{N \,.\, M_{ii}{}^2}$$

or

$$\mathsf{E}\,(\varrho_{i|k;1\ldots n}) = \frac{1}{M_{ii}} \sqrt{\frac{M\,.\,M_{ii,\,kk}}{N}} \quad \ldots\ldots \quad (18^B)$$

Integrating along u_α $(\alpha \neq i, \alpha \neq k)$ we obtain a coefficient of regression of order $n-3$:

$$\left.\begin{aligned} \varrho_{i|k;1\ldots,(\alpha)\ldots n} &= -\frac{p_{ik}^{(\alpha)}}{p_{ii}^{(\alpha)}} = -\frac{\lambda_{ik}^{(\alpha)}\,h_k^{(\alpha)}}{h_i^{(\alpha)}} = -\frac{M_{\alpha\alpha,\,ik}}{M_{\alpha\alpha,ii}} = \\ &= \gamma_{ik;1\ldots(\alpha)\ldots n} \cdot \frac{\varepsilon_{i;1\ldots(\alpha)\ldots n}}{\varepsilon_{k;1\ldots(\alpha)\ldots n}}, \end{aligned}\right\} (18^A)'$$

with

$$\mathsf{E}\,(\varrho_{i|k;1\ldots(\alpha)\ldots n}) = \frac{1}{M_{\alpha\alpha,\,ii}} \sqrt{\frac{M_{\alpha\alpha}\,.\,M_{\alpha\alpha,\,ii,\,kk}}{N}} \quad \ldots \quad (18^B)'$$

Likewise, integrating along u_α and u_β (α and β different from i and k) we arrive at a coefficient of regression of order $n-4$:

$$\varrho_{i|k;1\ldots(\alpha\beta)\ldots n} = -\frac{M_{\alpha\alpha,\,\beta\beta,\,ik}}{M_{\alpha\alpha,\,\beta\beta,\,ii}} = \gamma_{ik;1\ldots(\alpha\beta)\ldots n} \cdot \frac{\varepsilon_{i;1\ldots(\alpha\beta)\ldots n}}{\varepsilon_{k;1\ldots(\alpha\beta)\ldots n}}, \quad . \quad (18^A)''$$

with

$$\mathsf{E}\,(\varrho_{i|k;1\ldots(\alpha\beta)\ldots n}) = \frac{1}{M_{\alpha\alpha,\,\beta\beta,ii}} \sqrt{\frac{M_{\alpha\alpha,\beta\beta}\,.\,M_{\alpha\alpha,\,\beta\beta,\,ii,\,kk}}{N}} \quad .\;. \quad (18^B)''$$

Finally we retain in $M_{\alpha\alpha,\beta\beta,\gamma\gamma,\ldots}$ only $m_{ii}\,m_{kk} - m_{ik}{}^2$, in $M_{\alpha\alpha,\beta\beta,\gamma\gamma,\ldots ii}$ only m_{kk}, in $M_{\alpha\alpha,\beta\beta,\gamma\gamma,\ldots ik}$ only $-m_{ik}$, whilst $M_{\alpha\alpha,\beta\beta,\gamma\gamma,\ldots ii,kk}$ passes into 1.

So we find for the coefficient of regression of order 2:

$$\varrho_{i|k} = \frac{+\,m_{ik}}{m_{kk}} = \gamma_{ik}\frac{\varepsilon_i}{\varepsilon_k} \quad \ldots\ldots\ldots \quad (19^A)$$

with

$$\mathsf{E}\,(\varrho_{i|k}) = \frac{1}{m_{kk}} \sqrt{\frac{m_{ii}\,m_{kk} - m_{ik}{}^2}{N}} = \frac{\varepsilon_i}{\varepsilon_k} \sqrt{\frac{1-\gamma_{ik}{}^2}{N}} \quad \ldots \quad (19^B)$$

The correlative relation between two parameters q and r is given by their coefficient of correlation

$$\gamma\,(q,r) = \frac{\overline{\triangle q\,\triangle r}}{\mathsf{E}\,(q)\,.\,\mathsf{E}\,(r)}\,.$$

Now the parameters (such as p_{kl}, m_{kl}, $\gamma_{kl;1\ldots(\alpha\beta)\ldots n}$, $\varrho_{i|k;1\ldots(\alpha\beta)\ldots n}$ etc.), being simple functions either of the magnitudes p_{kl} (and P), or of the magnitudes m_{kl} (and M), or, eventually, of M_{kl} or P_{kl}, and the values of the mean products $\overline{\triangle p_{ij}\triangle p_{kl}}$ etc. being already determined, we can compute $\overline{\triangle q\,\triangle r}$ without trouble; moreover the values of $\mathsf{E}(q)$ (resp. $\mathsf{E}(r)$), have already been calculated precedently.

So the calculation of $\gamma\,(q,r)$ does not give rise to new difficulties.